



San Diego Unified School District

Instructional Module to Enhance the Teaching of

HARCOURT

Math

California Edition

Grade 3

Included: Grade Three Order of Units and Pacing Guide; Mathematics Curriculum Map - Grade 3

Module 2 - Revised
*Multiplication Concepts and
Facts*

- WORK IN PROGRESS -

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MODULE 2 – MULTIPLICATION CONCEPTS AND FACTS

Modules represent individual units of study that lead to essential learnings

THREADS THROUGHOUT THE YEAR:

The threads represent ongoing learning opportunities in which students should be actively engaged throughout all units of inquiry during the entire school year. These items should not be isolated to any one particular unit of inquiry.

Students will:

- Develop understanding of numbers and the number system and use their understanding to solve problems and recognize reasonable results.
- Develop understanding of and fluency in basic computation and procedural skills.
- Use mathematical reasoning to solve problems.
- Communicate their mathematical thinking by using words, numbers, symbols, graphs and charts.
- Use equations and variables to express generalizations of patterns and relationships.
- Develop logical thinking to analyze evidence and build arguments to support or refute a hypothesis.
- Make connections among mathematical ideas and between other disciplines.
- Develop and use strategies, skills, and concepts to solve problems.
- Use appropriate tools, including technology, as vehicles to learn mathematical concepts.

These are essential learnings that represent bigger ideas/concepts:

- Students understand that multiplication involves counting groups of like size and determining the total (multiplicative thinking).
- Students understand the inverse relationship between multiplication and division and relate it to missing factors in multiplication when the product and one factor are given.
- Students use appropriate models to solve problems and explain the properties of multiplication.
- Students understand conceptual relationships between facts and use known facts to figure out unknown facts.

These are essential questions that learners ask themselves in order to achieve the essential learnings:

- How do I model and explain the Commutative Property, the Zero Property and Property of 1, and then use the properties when solving multiplication situations?
- **What patterns do I notice when I model multiplication as “skip counting” with repeated addition using factors from 1-10?*
- **What strategies do I use to understand, practice, and learn multiplication facts?*
- How do I find the “missing” factor when given the product and one factor?
- **How do I use arrays to model and solve multiplication problems?*
- How do I use a table to record a number pattern and then write a rule to describe the pattern?
- How do I model and record multiplication with 3 factors using the Associative Property of Multiplication?

*** Presented in previous grades**

Resources: Van de Walle: Chapter 10 (pp.143-154); Mathematics Source Book: Multiplication (pp. 37-47)

Module 2: MULTIPLICATION CONCEPTS & FACTS

Unit 2: Chapters 7 - 10

5 WEEKS

Key Mathematics Concepts:

- Understand **multiplication** as repeated addition-
- Understand and represent **multiplication** as "equal groups of" - Use "models" to connect to real world situations such as # of days in 8 weeks = 7 (number of days in a week) X 8 (weeks)-
- Use arrays as a model of **multiplication** along with skip counting/number patterns-
- Learn "multiplication facts" through strategies/number patterns-
- Connect **multiplication** to the concept of **division**-
- Understand and use **multiplication properties** to solve problems:
Commutative, Distributive and Associative Properties-
Identity Property: Multiplying by 1, by 10, by 100 and by 0-

<p>Chapter 7 <u>Understand Multiplication</u></p> <p>Lesson 1: Algebra: Connect Addition and Multiplication</p> <p>Lesson 2: Multiply by 2's and 5's</p> <p>Lesson 3: Hands On: Arrays</p> <p>Lesson 4: Multiply by 3's</p> <p>Lesson 5: Problem Solving Skill: <i>Too Much/Too Little Information</i></p>	<p>Chapter 8 <u>Multiplication Facts Through 5</u></p> <p>Lesson 1: Multiply by 0's and 1's</p> <p>Lesson 2: Multiply by 4's</p> <p>Lesson 3: Problem Solving Strategy: <i>Find a Pattern</i></p> <p>Lesson 4: Practice Multiplication</p> <p>Lesson 5: Algebra: Find Missing Factors</p>
<p>Chapter 9 <u>Multiplication Facts & Strategies</u></p> <p>Lesson 1: Multiply by 6's</p> <p>Lesson 2: Multiply by 7's</p> <p>Lesson 3: Multiply by 8's</p> <p>Lesson 4: Problem Solving Strategy: <u>Draw a Picture</u></p> <p>Lesson 5: Algebra: Practice the Facts</p>	<p>Chapter 10 <u>Multiplication Facts & Patterns</u></p> <p>Lesson 1: Multiply by 9's and 10's</p> <p>Lesson 2: Algebra: Find the Rule</p> <p>Lesson 3: Algebra: Multiply with 3's Factors</p> <p>Lesson 4: Problem Solving Skill: Multi-step Problems</p>

GRADE 3

Harcourt Mathematics

UNIT 3: MULTIPLICATION CONCEPTS AND FACTS

Module 2: 5 Weeks of Instruction



<p>Day 1: Chapter 7: Understand Multiplication</p> <p>Lesson 7.3 Hands on Arrays</p> <p>Day 6: Chapter 8: Multiplication Facts Through 5</p> <p>Lesson 8.3 Problem Solving Strategy: "Find a Pattern"</p> <p>Day 11: Chapter 9: Multiplication Facts and Strategies</p> <p>Lesson 9.4 Problem Solving Strategy: "Draw a Picture"</p> <p>Day 16: Chapter 9 Review</p> <p>Chapter 10: • Intro: Multiplication Facts and Patterns • Problem Solving Project</p> <p>Day 21: Unit 3 Review</p> <p>• Math Detective • Challenge</p>	<p>Day 2: Lesson 7.1 Algebra: Connect Addition & Multiplication</p> <p>Day 7: •Performance Assessment: Class Play</p> <p>Lesson 8.1 Multiply with 0 & 1</p> <p>Day 12: Lesson 9.1 Multiply with 6</p> <p>Day 17: Lesson 10.2 Algebra: Find the Rule</p> <p>Day 22: California Connections</p>	<p>Day 3: Lesson 7.2 Multiply with 2 & 5</p> <p>Day 8: Lesson 8.2 Multiply with 4</p> <p>Day 13: Lesson 9.2 Multiply with 7</p> <p>Day 18: Lesson 10.1 Multiply with 9 & 10</p> <p>Day 23: Multiplication Review (Module does not have lesson for Day 23.)</p>	<p>Day 4: Lesson 7.4 Multiply with 3 • Thinker's Corner</p> <p>Day 9: Lesson 8.4 Practice Multiplication • Thinker's Corner</p> <p>Day 14: Lesson 9.3 Multiply with 8</p> <p>Day 19: Lesson 10.3 Algebra: Multiply with 3 Factors</p> <p>Day 24: Performance Assessments: Class Play and A New Game (See Performance Assessment Booklet)</p>	<p>Day 5: Lesson 7.5 Problem Solving: "Too Much/Too Little Information"</p> <p>Day 10: Lesson 8.5 Algebra: Find Missing Factors</p> <p>Day 15: Lesson 9.5 Algebra: Practice the Facts • Thinker's Corner</p> <p>Day 20: Lesson 10.4 Problem Solving Skill: "Multi-step Problems"</p> <p>Day 25: Unit 3: Assessment Introduce: Unit 1 Understand Numbers & Operations</p>
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UNIT 3: MULTIPLICATION CONCEPTS AND FACTS

Module 2: Chapter 7

DAY 1: LESSON: 7.3, pp. 120 - 121

MATERIALS:	*Transparency 7.3; Color tiles: 1 bag of 50 tiles for each group of 2 -4 students; Cm grid paper
LESSON FOCUS:	Hands On: Arrays
CALIFORNIA STANDARDS:	Algebra and Functions: 1.5 Number Sense: 1.0, 1.1, 1.5 *2.2: Memorize to automaticity the multiplication table for numbers between 1 and 10. Mathematical Reasoning: 1.1, 2.0, 2.3, 3.0
Purpose of Lesson:	<p>To understand how to use "arrays" and the Commutative Property (called the Order Property in Harcourt) to explore the concept of multiplication.</p> <ul style="list-style-type: none"> -Multiplication sentence can be represented by rectangular arrays with equal amounts in each row and each column -The Commutative Property states: $a \times b = c$ AND $b \times a = c$ OR $a \times b = b \times a$ -Quantities that can be represented in square arrays (the same number of rows and columns) are identified as perfect squares. <p>(Lesson Note: If this is the students' first experience using manipulatives - ex: color tiles - they will need a few minutes for "free exploration" and to discuss the "expectations" for using the materials.)</p>
<p>LAUNCH:</p> <p>*Transparency 7.3</p> <p>Orient students to concepts</p> <p>Color tiles: 1 bag of 50 for each group of 2 -4 students</p>	<p>*Warm up: PG. 120A: Problem of the Day</p> <p>Discussion Question: What do you know about multiplication? Chart students' responses.</p> <p>Question: I have 6 tiles. What are all the different ways I can arrange them as a rectangle? Students show and discuss arrangements. Name arrangements as "arrays". All rows are the same length and all columns have the same number. Use multiplication notation when recording the arrays. Example: 4 by 3 array is 4×3.</p> <p>Question: How do we know if we have found all the arrangements? Think by yourself and then discuss with partner.</p>

	<p>Discuss with students: $1 \times 6 =$  is equivalent to $6 \times 1 =$ </p> <p>2 arrays are congruent if they can be rotated/flipped and cover each other array exactly.</p> <p>This is called the Commutative Property (Harcourt calls it: Order Property - students should learn the term: Commutative).</p>
<p>EXPLORE: Work with the concept. Focus on student's "doing mathematics." Color Tiles Cm grid paper</p>	<p>Task: Find all the arrays using 4 - 12 tiles. Students work with partners/group of 4 to find and record all the different arrays on grid paper. Students share results - discuss what they noticed. Which numbers have only 1 array? (prime) Why do some numbers have more arrays than others? (composite; # of factors) Is there anything interesting that you notice about the arrays? How can you predict the number of arrays for any number? Explain how you decided that you had found all the arrays for any number?</p> <p>Post the arrangements for the arrays for 4 - 12. On each note the corresponding multiplication facts and product.</p>
<p>PRACTICE: Color Tiles Cm grid paper</p>	<p>Find all the arrays using: 9, 15, 18 and 20 tiles Record results. Students check results with classmates. Students need to note multiplication facts and product for each array.</p>
<p>SUMMARIZE/ CLOSURE: Connect purpose to activities</p>	<p>With students: Label the arrays for 18 and 20. Discuss: Why do these 2 numbers have 3 different arrays? Why does 15 have only 2? How many arrays do they think there are the number for 16? Explain reasoning/thinking. Find all the arrays for 16. Discuss and chart with class: What is an array? What mathematical information does it give? (Encourage student to use terms such as: factors, product, prime and composite numbers in explanations.)</p>

HOMEWORK: 2 pieces of cm grid paper per student	Find all the arrays using 24 tiles. Ask students to predict the number of arrays they think there are for 24. Record the results on grid paper and label each with the factors and product. Explain how you know that you have found all the different arrays.
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ROUTINES:

Use "Hundred Grid/Chart" to skip count to 100 by different numbers.

Ask students to predict which "skip counting" numbers will land on 100 and explain their reasoning.

If you can mark the grid/chart, discuss the different skip counting patterns for different numbers.

See: Number Grid Notes: Next page.

NUMBER GRID NOTES

Number grids are also useful for finding the difference between two numbers. For example, to find the difference between 84 and 37, you could start at 37, count the number of tens going down to 77 (4 tens, or 40), and then count the ones going from 77 to 84 (7 ones or 7). The difference between 84 and 37 is, thus, 4 tens and 7 ones, or 47. This difference is sometimes referred to as the distance between the points 37 and 84 on a number line (or grid).

Number Grid Activities

Skip Counting:

1. Start at 1 and count by 4s on the number grid. Color (or mark an x through) each number in your count. What patterns do you notice?
2. Start at 1 and count by 5s on the number grid. Color (or circle) each number in your count. What patterns do you notice?
3. What other number patterns can you find?

Comparing Numbers on the Hundred Grid:

4. Use the number grid to help you solve these problems:
 - a. Which is less, 83 or 43? How much less?
 - b. Which is more, 90 or 55? How much more?
 - c. Find the difference between 71 and 92.
Explain your strategy for finding the difference between the 2 numbers.
 - d. Find the difference between 26 and 46.
What strategy did you use to find the difference between the 2 numbers?
 - e. Select two amounts on the number grid. What is the difference between these amounts? Describe your strategy for finding the difference.
5. Show an incomplete grid, point to a square on the grid. Ask: This square represents what number? How do you know? What was your strategy?
6. Which digit is used the greatest number of times on the number grid?
How do you know?
9. Which digit is used the least number of times on the number grid?
How do you know?
10. Is the 6 used more times in the ones place or in the tens place?
Why do you think that is?
11. What questions do you have that the grid could help you answer?

UNIT 3: MULTIPLICATION CONCEPTS AND FACTS
Module 2: Chapter 7

DAY 2: LESSON: 7.1, pp. 116 - 117

MATERIALS:	Number line or Hundred Grid; Bags of color tiles
LESSON FOCUS:	Algebra: Connect Addition and Multiplication
CALIFORNIA STANDARDS:	Number Sense: 2.0, 2.8 *2.2: Memorize to automaticity the multiplication table for numbers between 1 and 10. Algebra and Functions: 1.0 Mathematical Reasoning: 1.1, 2.3, 3.0, 3.2
Purpose of Lesson:	To understand the connections between repeated addition and multiplication. - Multiplication only connects directly to addition when groups of <u>equal</u> size are added.
LAUNCH: Orient students to concepts Number Line/Hundred Grid	HW: Share solutions and discuss. Students share with table group the results of HW- Discuss strategies for knowing when all the arrays for a quantity have been found. What did they notice about the number of different arrays for 24? Skip counting: Use Number Line/Grid to model "skip counting" Pg. 116: Learn - "Slurp" Question- Pg. 116 B: Use: English Language Learners strategy - to answer question with students Model sketch on the board/overhead. Label and discuss strategies for finding the total. Highlight: Pg. 116: "Math Idea"
EXPLORE: Work with the concept. Focus on student's "doing mathematics." Bags of color tiles for small group work	Pg. 116: Do "Check" with students. Have them model with color tiles, record and label with addition and multiplication sentence. Put problems on board: "3 groups of 6" "4 groups of 5" Students show groups of, arrays and number sentences. With partner, choose 1 problem and write story problem. Discuss and share results & strategies for solving problems. Students match story problem to representation.

PRACTICE: Focus on Communication and Representation	Give partners/tables 16 tiles. Task: Show all the arrays and "groups of" for 16 Write multiplication and addition sentences for each Note: Pg. 117: Common Error Alert
SUMMARIZE/ CLOSURE: Connect purpose to activities	Pg. 117: Assess: Discuss Ask students: How are the 2 solutions alike? Different? Explain why both are acceptable for the situation. Pg. 117: Assess: Write Students respond individually to question. This is good "journal" prompt that could be used throughout the unit.
HOMEWORK:	Pg. 116: Problems 6 - 11 Show and explain solution to problem 11

ROUTINES:

Skip Counting: Continue skip-counting practice. Choose 2 numbers and record the pattern.
 Ex: Count by 4s and 6s

4	8	12	16	20	24	28	32
8	16	24	32	40	48	56	64

What do you notice about the pattern? How would knowing these patterns help you learn the multiplication facts?

If you skip count by 4 and 8, will you land exactly on 100? Explain your reasoning.

UNIT 3: MULTIPLICATION CONCEPTS AND FACTS

Module 2: Chapter 7

DAY 3: LESSON: 7.2, pp. 118 - 119

MATERIAL:	Color tiles: 1 bag for each table; Practice 7.2 - 1 copy per student
LESSON FOCUS:	Multiply with 2 and 5
CALIFORNIA STANDARDS:	Number Sense: 2.0, 2.8 *2.2: Memorize to automaticity the multiplication table for numbers between 1 and 10. Algebra and Functions: 1.0, 2.1 Mathematical Reasoning: 1.0, 2.2, 2.3, 2.4
Purpose of Lesson:	Understand multiplication by 2s and 5s. - Multiplication by 2 always gives a product that is even. - Multiplication by 5s always gives a product with a 0 or 5 in the ones place.
LAUNCH: Orient students to concepts Color Tiles: 1 bag for each table	HW: Partner check Warm up: Skip count by 2's to 20 and 5s to 50. Sketch a number line on board/overhead to show counting patterns. Discuss with partner what you notice about these patterns. Pg. 118: Learn: "Smart Rocks" question. Read to students. Students use color tiles to model the problem. Pg. 118: "Guided Instruction" questions - students explain/justify answers. Highlight: Math Idea: "factors" and "product" - add to word wall and Use equation to "model"
EXPLORE: Work with the concept. Focus on student's "doing mathematics."	Pg. 118: "Check": Do with students - identify: factors and product Pg. 119: Problem 25: Students model, write number sentence and identify factors and product. Pg. 119: Problems 10 & 11: Students write a problem for each equation and make representation/model.
PRACTICE: Focus on Communication and Representation	Pg. 119: Problems: 23 - 24 and 26 -27: Task: Model/represent problem and explain/justify answer. Discuss solutions and strategies for solving.
SUMMARIZE/ CLOSURE:	Pg. 119: Assess: Discuss and chart student responses. Discuss students strategies for learning facts for 2s and 5s Record suggestions.
HOMEWORK:	Practice 7.2

