



San Diego Unified School District

Instructional Module to Enhance the Teaching of

H A R C O U R T

Math

California Edition

Grade 3

Module 5: Revised

Division Concepts and Facts

—WORK IN PROGRESS—

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San Diego City Schools
Instruction and Curriculum Division
GRADE 3 – MATHEMATICS CURRICULUM MAP

MODULE 5 – DIVISION CONCEPTS AND FACTS

Modules represent individual units of study that lead to essential learnings

THREADS THROUGHOUT THE YEAR:

The threads represent ongoing learning opportunities in which students should be actively engaged throughout all units of inquiry during the entire school year. These items should not be isolated to any one particular unit of inquiry.

- Students will:
- Develop understanding of numbers and the number system and use their understanding to solve problems and recognize reasonable results.
 - Develop understanding of and fluency in basic computation and procedural skills.
 - Use mathematical reasoning to solve problems.
 - Communicate their mathematical thinking by using words, numbers, symbols, graphs and charts.
 - Use equations and variables to express generalizations of patterns and relationships.
 - Develop logical thinking to analyze evidence and build arguments to support or refute a hypothesis.
 - Make connections among mathematical ideas and between other disciplines.
 - Develop and use strategies, skills, and concepts to solve problems.
 - Use appropriate tools, including technology, as vehicles to learn mathematical concepts.

These are essential learnings that represent bigger ideas/concepts:

- *Students understand the inverse relationship between multiplication and division and use it to solve, explain and check division problems.*
- *Students understand how repeated subtraction can be used to solve division problems.*
- *Students use models to explain meanings of the operations and their properties.*
- *Students translate between the different representations of solutions and situations involving multiplication and division.*

These are essential questions that learners ask themselves in order to achieve the essential learnings:

- How do I model and use the inverse relationship of multiplication and division with arrays* and number facts?
- How do I use the inverse relationships of multiplication and division to solve and check division problems?
- How do I use understanding of subtraction to make sense of the meaning of division?
- How do I use concrete materials and the context to explain the two models of division (equal groups of and sharing)?
- How do I model a problem situation and translate the problem into a number sentence (equation) with the correct operational symbols?
- How do I use multiplication facts and models to solve and explain division problems with single digit divisors?
- How do I explain and use the properties of multiplying and dividing by one and multiplying by zero to solve problems?
- How do I write expressions that represent situations involving division?
- How do I select operational symbols to make number sentences true involving division?

** Presented in previous grades*

Resources: Van de Walle: Chapter 10 & 11 (pp. 143-152 and pp173-174); *Mathematics Source Book: Division* (pp.48-58)

UNIT 4: DIVISION CONCEPTS AND FACTS**Key Mathematical Concepts:**

- Understand and use the two models of division: equal groups of and sharing - how many in each group.
- Understand how to compute and solve problems involving addition, subtraction, multiplication, and division using accurate and efficient strategies.
- Understand and use the inverse relationships of multiplication and division and use this relationship to compute and check results.
- Understand and use the special properties of 0 and 1 in multiplication and division.
- Understand how to represent relationships of quantities using mathematical expressions, equations, or inequalities.
- Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models to explain mathematical reasoning.
- Represent division situations using words, numbers, manipulatives and representations.

<p>Chapter 11: <u>Understand Division</u></p> <p>Lesson 11.1: Hands On: The Meaning of Division</p> <p>Lesson 11.2: Relate Subtraction and Division</p> <p>Lesson 11.3: Algebra: Relate Multiplication and Division</p> <p>Lesson 11.4: Algebra: Fact Families</p> <p>Lesson 11.5: Problem Solving Strategy: Write a Number Sentence</p>	<p>Chapter 12: <u>Division Facts Through 5</u></p> <p>Lesson 12.1: Divide by 2 and 5</p> <p>Lesson 12.2: Divide by 3 and 4</p> <p>Lesson 12.3: Divide with 0 and 1</p> <p>Lesson 12.4: Algebra: Write Expressions</p> <p>Lesson 12.5: Problem Solving Skill: Choose the Operation</p>
<p>Chapter 13: <u>Division Facts Through 10</u></p> <p>Lesson 13.1 Divide by 6, 7, and 8</p> <p>Lesson 13.2 Divide by 9 and 10</p> <p>Lesson 13.3 Practice Division Facts Through 10</p> <p>Lesson 13.4 Algebra: Find the Cost</p> <p>Lesson 13.5 Problem Solving Strategy: Work Backward</p>	

Unit 4: DIVISION CONCEPTS AND FACTS

MODULE 5 NOTES

- Lessons with an “ * ” can be optional if additional instructional time is needed for particular unit concepts/topics.

Several of the “* lessons” provide important experiences with problem-solving strategies and can be built into other lessons.

- Students may need additional experiences working with large numbers and the operations. This on-going focus can be built into Routines.
- The focused “Problem Solving” lessons often provide a strong foundation for the concepts of the chapter and can be taught as the first in the chapter’s sequence. This provides a context for the concepts that follow.
- This module does not have lesson plans for:
 - Day 18: California Connection
 - Day 19: Unit Review and Assessment
- Whenever possible, students should be asked how they thought about the problems and to explain their thinking.

**GRADE 3
MODULE 5**

Unit 4 – 20 DAYS

<p><u>Day 1</u> CHAPTER 11</p> <p>Lesson 11.1: Hands On: The Meaning of Division</p>	<p><u>Day 2</u></p> <p>Lesson 11.2: Relate Subtraction and Division</p>	<p><u>Day 3</u></p> <p>Lesson 11.3: Algebra: Relate Multiplication and Division</p>	<p><u>Day 4</u></p> <p>Lesson 11.4: Algebra: Fact Families – Day 1 of 2 Day Lesson</p>	<p><u>Day 5</u></p> <p>Lesson 11.4: Algebra: Fact Families – Day 2 of 2 Day Lesson</p>
<p><u>Day 6</u></p> <p>Lesson 11.5: Problem Solving Strategy: Write a Number Sentence</p>	<p><u>Day 7</u> CHAPTER 12</p> <p>Lesson 12.1: Divide by 2 and 5</p>	<p><u>Day 8</u></p> <p>Lesson 12.2: Divide by 3 and 4</p>	<p><u>Day 9</u></p> <p>Lesson 12.3: Divide with 0 and 1</p>	<p><u>Day 10</u></p> <p>Lesson 12.4: Algebra: Write Expressions</p>
<p><u>Day 11</u></p> <p>Lesson 12.5: Problem Solving Skill: Choose the Operation</p>	<p><u>Day 12</u> CHAPTER 13</p> <p>Lesson 13.1: Divide by 6, 7, and 8 – Day 1 of 2 Day Lesson</p>	<p><u>Day 13</u></p> <p>Lesson 13.1: Divide by 6, 7, and 8 – Day 2 of 2 Day Lesson</p>	<p><u>Day 14</u></p> <p>Lesson 13.2: Divide by 9 and 10</p>	<p><u>Day 15</u></p> <p>Lesson 13.3: Practice Division Facts Through 10</p>
<p><u>Day 16</u></p> <p>Lesson 13.4: Algebra: Find the Cost</p>	<p><u>Day 17</u></p> <p>Lesson 13.5: Problem Solving Strategy: Work Backward</p>	<p><u>Day 18</u></p> <p>California Connection</p>	<p><u>Day 19</u></p> <p>Unit Review and Assessment</p>	<p>Introduce Next Unit</p>

DAY: 1

Unit 4: DIVISION CONCEPTS AND FACTS

Module 5: Chapter 11: Understand Division

LESSON 11.1, pg. 184-185

MATERIALS:	30 counters for per student
LESSON FOCUS:	Hands On: The Meaning of Division
CALIFORNIA STANDARDS:	<p>Number Sense 2.0: Students calculate and solve problems involving addition, subtraction, multiplication, and division.</p> <p>Mathematical Reasoning 2.3: Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models to explain mathematical reasoning.</p> <p>Mathematical Reasoning: 1.0, 3.2</p>
PURPOSE OF LESSON:	Understand division as the inverse operation to multiplication. Understand and recognize division one model of division as finding the equal number of items in each group and another model as finding the number of equal groups.
<p>LAUNCH: Introduce students to concepts.</p> <p>Make counters available.</p>	<p>Ask students:</p> <ul style="list-style-type: none"> • To count the number of students in class. • Count, e.g., 16, and begin table below. • Fill in data for each step as it is done. (Note: If total is an odd number, have the class include the teacher in the count.) • Discuss what it means to have equal groups. • <i>How can we divide 16 (or whatever # is in class) into 2 equal groups?</i> • Students may predict and then justify their responses with manipulatives or drawings. • Record data in table and discuss as you go. • <i>How many in each group? (8)? Discuss.</i> • <i>Can we divide (or separate or distribute) 16 into equal groups in any other way?</i>
EXPLORE: Work with the concept. Focus on students “doing mathematics.”	<p>Try It, Pg. 184:</p> <ul style="list-style-type: none"> • Do with students, working in pairs. <p>Connect, Pg. 185;</p> <ul style="list-style-type: none"> • Ask pairs of students to discuss the different representations of the problem.

PRACTICE: Focus on Communication and Representation.	Practice Pg. 185, #5-8. Discuss Total Number of Students Number of Equal Groups Number in Each Group											
	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 33%; text-align: center;">16</td> <td style="width: 33%; text-align: center;">2</td> <td style="width: 33%; text-align: center;">8</td> </tr> <tr> <td style="text-align: center;">16</td> <td style="text-align: center;">4</td> <td></td> </tr> <tr> <td style="text-align: center;">16</td> <td style="text-align: center;">8</td> <td></td> </tr> <tr> <td style="text-align: center;">16</td> <td style="text-align: center;">16</td> <td></td> </tr> </table> <ul style="list-style-type: none"> • Discuss data in table. • Continue dividing 16 into equal groups. Be sure to include 1 group of 16 and 16 equal groups each having 1 student. • Discuss data in completed table. Have each student make up a question based on the table for the class to answer. • Record all student responses. Students justify with counters or drawings. <p>Explore, Pg. 184. Teach, Pg. 184, Guided Instruction questions to guide discussion.</p>	16	2	8	16	4		16	8		16	16
16	2	8										
16	4											
16	8											
16	16											
SUMMARIZE: Connect purpose to activities.	<ul style="list-style-type: none"> • Discuss dividing things into equal groups. • Help them make the distinction between dividing them into groups vs. dividing into EQUAL groups; for example 8 divided into 2 <u>equal</u> groups or 8 divided into 2 groups (7 and 1; 5 and 3, etc.). • Students make up, write, and solve a word problem involving dividing something into equal groups. • Share students' problems. 											
HOMEWORK:	Pg. 185: Practice #1-4. Draw chart and pictures of counters to represent each problem. Mixed Review and Test Prep. Pg. 185											

ROUTINES:

Mental Arithmetic: Skip Counting by 2, 3 & 5,
 Ask students to skip count from 1 to 100, first by 2, then 3, then 5.
 Then ask students to skip count backwards from 1 to 100 by 5, then 2, then 3.
 This exercise helps students become proficient with repeated subtraction.

Record skip counting patterns on board/overhead for additional support if needed.

Reteach/reinforce any skills and concepts identified in homework check as needing attention.

DAY: 2
 Unit 4: DIVISION CONCEPTS AND FACTS
 Module 5: Chapter 11: Understand Division
 LESSON 11.2 pg. 186-187

MATERIALS:	16 counters for each student; Number line for each student, Pg. TR5 Calculators for pairs
LESSON FOCUS:	Relate Subtraction to Division
CALIFORNIA STANDARDS:	Mathematical Reasoning: 1.0, 2.0, 2.2, 2.4, 3.0, 3.2 2.3: Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models to explain mathematical reasoning. Number Sense: 2.8 2.0: Students calculate and solve problems involving addition, subtraction, multiplication, and division. Algebra and Functions: 1.0, 1.1, 1.3
PURPOSE OF LESSON:	Understand division as repeated subtraction of equal groups (the number of times the equal group is subtracted represents the quotient); Connect repeated subtraction as skip counting backwards.
LAUNCH: Introduce students to concepts. 16 Counters for each student	<ul style="list-style-type: none"> • Give each student 16 counters. <p>Technology: Calculator, Pg. 186B; then, Alternative Teaching Strategy, Pg. 186B</p> <ul style="list-style-type: none"> • Discuss. Help students understand/recognize that division can be seen as repeated subtraction. (separating/removing equal groups) • Continue by starting with 12 counters and repeat the procedure of separating into equal groups. • Remind students to count the number of times they subtract 3 from 12 to get 0 (4); so there are four (equal) groups of 3 in 12.
EXPLORE: Work with the concept. Focus on students “doing mathematics.”	<p>Learn, Pg. 186: Get in the Game.</p> <ul style="list-style-type: none"> • Write problem on the board/overhead. • Ask students to solve using their counters and record their work in numbers/words/diagrams. • Discuss strategies and solutions. <p>Highlight: Math Idea, Pg. 186.</p> <ul style="list-style-type: none"> • Discuss.
PRACTICE: Focus on Communication and Representation.	Pg. 187: Check: Do #s: 1 – 3 with students. Practice & Problem Solving, Pg. 187 #14 – 20. Students work with partners. Share & discuss solutions.

SUMMARIZE: Connect purpose to activities.	ASSESS, TE Pg. 187: DISCUSS: Do with students. • Students to record their responses in their journals. ASSESS, TE Pg. 187: WRITE: • Summarizing that division can be seen as repeated subtraction.
HOMEWORK:	Mixed Review & Test Prep, Pg. 187.

ROUTINES:**Let's Buy Them All**

Explain that you will be showing the students how much a given item costs; e.g., 25¢. Then ask them how much it would cost to buy multiples of this item.

List items such as:

Ball:	\$1.10	How much for 2, 4 or 5?
Candy:	6¢	How much for 6, 10 or 100?
Pencil:	12¢	How much for 2, 3 or 4?
Eraser:	7¢	How much for 2, 3 or 10?
Small Doll:	50¢	How much for 2, 3, 4, 5 or 10?

Reteach/reinforce any skills and concepts identified in homework check needing attention.

DAY: 3
 Unit 4: DIVISION CONCEPTS AND FACTS
 Module 5: Chapter 11: Understand Division
 LESSON 11.3 Pg. 188-189

MATERIALS:	10 counters for each student; Routines: 1 copy of Performance Assessment: 34A: Daisy Garden for each student;
LESSON FOCUS:	Algebra: Relate Multiplication and Division
CALIFORNIA STANDARDS:	Number Sense: 2.2, 2.8 2.0: Students calculate and solve problems involving addition, subtraction, multiplication, and division. 2.3: Use the inverse relationship of multiplication and division to compute and check results. Algebra and Functions: 1.0, 1.1, 1.2 Mathematical Reasoning: 1.0, 3.2
PURPOSE OF LESSON:	Understand and use the inverse relationship of multiplication and division through practice with arrays and number facts.
LAUNCH: Introduce students to concepts. Counters for each student.	<ul style="list-style-type: none"> • Give each student 10 counters. • Ask students to make a rectangle with their counters. Remind students that this is an array. • Ask students to record a multiplication sentence that represents what you just modeled. • Have students share the different arrays. (1x10; 10x1; 2x5; 5x2) Discuss the number of counters in each row/column, depending on the orientation. • If necessary, help students begin “at the end” or work backward, by starting with 5 (rows), multiplying by the number in each row (2) and getting 10; $5 \times 2 = 10$ or $2 \times 5 = 10$, depending on the orientation of the students’ array. <p>See Alternative Teaching Strategy, Pg. 186B guide writing of multiplication/division sentences.</p> <ul style="list-style-type: none"> • Emphasize the inverse relationship of multiplication and division.
EXPLORE: Work with the concept. Focus on students “doing mathematics.”	Learn, pg. 188: Stick with Stamps. Write problem on board/overhead. Teach, pg. TE 188: Guided Instruction questions to guide discussion. Practice & Problem Solving, Pg. 189 # 6 – 9. <ul style="list-style-type: none"> • Do with students. • Discuss solutions and strategies/thinking.
PRACTICE: Focus on Communication and Representation.	Practice & Problem Solving, Pg. 189 #12 – 16. <ul style="list-style-type: none"> • Students work w/partner or individually. • Discuss and share solutions and strategies for solving.

SUMMARIZE: Connect purpose to activities.	<ul style="list-style-type: none"> • Discuss how division and multiplication are related. • Students choose a basic fact to illustrate using an array. • Students write division and multiplication sentences to describe array (fact family), and share work. • Include discussion of ASSESS: DISCUSS, Pg. 189.
HOMEWORK:	Practice & Problem Solving, Pg. 189: #10 - 11 Mixed Review and Test Prep: # 17 – 21

ROUTINES:

Pgs. 32-34 in *Performance Assessment Booklet*

Pgs. PA28-30: **Performance Assessment 3.4A Daisy Garden**

Read problem with students. Answer the questions.

Be encouraging... motivate, guide...

Be clear. Reword any words or phrases; insert others as needed.

Be supportive. Assist students who need help.

Be fair. Allow students adequate time to do best work.

Be flexible. All students will not do task in same order/way.

Involve students in evaluating process.

Suggested completed performance task on pg. PA32.

Rubrics (3-point) are provided on pgs. PA33-34. These should be used for next day's *Routine* when Performance Assessment is reviewed.

Reteach/reinforce any skills and concepts identified in homework check needing attention.

DAY: 4
 Unit 4: DIVISION CONCEPTS AND FACTS
 Module 5: Chapter 11: Understand Division
 LESSON 11.4 Pg. 190-193

****Day 1 of a 2-Day Lesson****

MATERIALS:	12 or more square pieces of paper or card stock, tag, etc., cut in half diagonally (see illustration on pg. 190); 1 envelope/baggie per student for storage; Routines: *Transparencies of student work for Daisy Garden problem.
LESSON FOCUS:	Algebra: Fact Families
CALIFORNIA STANDARDS:	Number Sense: 2.2, 2.8 2.0: Students calculate and solve problems involving addition, subtraction, multiplication, and division. 2.3: Use the inverse relationship of multiplication and division to compute and check results. Algebra and Functions: 1.0, 1.1, 1.2 Mathematical Reasoning: 1.0, 2.0, 2.3, 3.0, 3.2
PURPOSE OF LESSON:	Understand how to use fact families as model of inverse operations – connecting multiplication and division.
LAUNCH: Introduce students to concepts.	<ul style="list-style-type: none"> • Write on board/overhead: $3 + 7 = 10$ and $7 + 3 = 10$. • Ask students what they remember about number sentences and fact families (addition/subtraction are inverse operations). • Ask students what the other two number sentences are for this fact family. • Ask students which number sentence is missing in the family: $9 + 8 = 17$; $17 - 8 = 9$; $8 + 9 = 17$ • Students identify and write the missing sentence. • Discuss the inverse relationship between addition and subtraction, and connect to the inverse relationship of multiplication and division. • Ask students to “think about” the inverse operation of multiplication and division and write the fact family for $6 \times 3 = 18$. • Students check fact family with a partner. If time permits, have students write the fact family for $5 \times 5 = 25$ and make the connection to families with only two number sentences. • Discuss the different number of sentences between the families for $6 \times 3 = 18$ and $5 \times 5 = 25$.
EXPLORE: Work with the concept. Focus on students “doing mathematics.”	<ul style="list-style-type: none"> • Explain that students will identify fact families for multiplication and division. <p>Learn: Fun Facts: Activity. Pg. 190. See Teach, pg. 190; Guided Instruction questions to guide discussion.</p> <p>Practice & Problem Solving, Pg. 192 #10 – 13 Discuss. <u>Students save fact cards for further use.</u></p>

PRACTICE: Focus on Communication and Representation.	Pg. 193: Thinker's Corner: <ul style="list-style-type: none"> • Discuss rules of the game. Students play game with triangle fact cards. Suggestion: Model the game with a student. • Discuss using Thinker's Corner questions, margin TE Pg. 193.
SUMMARIZE: Connect purpose to activities.	Pg. 193: ASSESS: DISCUSS: Do with students. <ul style="list-style-type: none"> • Ask other similar questions; e.g., use $48 \div 8$ and $48 \div 6$.
HOMEWORK:	Practice & Problem Solving, Pg. 192: #30 - 35.

ROUTINES:

Pg. PA28-30: Review of *Performance Assessment 3.4A, Daisy Garden*.
Pgs. 32-34: *Performance Assessment* booklet.

Involve students in evaluation of Performance Assessment, *Daisy Garden*, completed in previous day's lesson. Share sample rubrics on pgs. PA33-34, beginning with Level 3 rubric. Ask why students think paper scored 3 is better than others. Have them identify the best points while you record them. Ask if anything important has been left out. Continue with similar questions. Share two other rubrics, one at a time, asking appropriate questions. Ask how each paper could be improved.

Return papers to students. Ask how they might improve their work. Help them edit and/or rewrite, if desired. Ask students what they learned from this experience. Discuss.

Reteach/reinforce any skills and concepts identified in homework check as needing attention.

