



San Diego Unified School District

Instructional Module to Enhance the Teaching of

HARCOURT

Math

California Edition

Grade 4

Module 2 - Revised
Multiplication and Division
Facts

- WORK IN PROGRESS -

MODULE 2 – MULTIPLICATION AND DIVISION FACTS

Modules represent individual units of study that lead to the essential learnings

THREADS THROUGHOUT THE YEAR:

The threads represent ongoing learning opportunities in which students should be actively engaged throughout all units of inquiry during the entire school year. These items should not be isolated to any one particular unit of inquiry.

Students will:

- Develop understanding of numbers and the number system and use their understanding to solve problems and recognize reasonable results.
- Develop understanding of and fluency in basic computation and procedural skills.
- Use mathematical reasoning to solve problems.
- Communicate their mathematical thinking by using words, numbers, symbols, graphs and charts and translate between different representations.
- Use equations and variables to express generalizations of patterns and relationships.
- Develop logical thinking to analyze evidence and build arguments to support or refute a hypothesis.
- Make connections among mathematical ideas and between other disciplines.
- Develop and use strategies, skills, and concepts to solve problems.
- Use appropriate tools, including technology, as vehicles to learn mathematical concepts.

These are essential learnings that represent bigger ideas/concepts:

- Students understand that all basic facts are conceptually related and can use this understanding to figure out new or unknown facts using known facts.
- They use flexible methods of computation that require understanding of the meaning of the operations and their properties.
- They use the inverse relationship between multiplication and division to develop flexible methods of division.

These are essential questions that learners ask themselves in order to achieve the essential learnings:

- **How can I explain and model the inverse operations of multiplication and division using skip counting, the multiplication chart, and fact families?*
- **How do I use my understanding of the associative and commutative properties to understand and learn multiplication facts?*
- What are some strategies I can use to find quotients and products mentally?
- How do I place parentheses in an expression to ensure that the order of performing the operations matches the events in the problem?
- How do I evaluate an expression with parentheses?
- How do I find a pattern in an input/output table, and then identify and write a rule using both an equation and words?

**** Presented in previous grade(s)***

Resources: Van de Walle: Chapters 10 & 11 (pp. 143-153 & 168-174); *Mathematics Sourcebook: Multiplication & Division* (pp. 37-47 & 48-58)

Module 2: Multiplication and Division Facts

3 WEEKS

Key Mathematics Concepts:

- Understand the relationship between multiplication and division-
- Understand multiplication as "repeated addition" and division as "repeated subtraction-
- Recognize division as the inverse of multiplication-
- Understand and use "strategies" to find products and quotients mentally-
- Understand and use the Commutative (called the Order Property in Harcourt) and the Associative Properties to understand and know multiplication and division facts-
- Understand parentheses and use them for grouping; recognize that with more than 1 operation-completing them in different order will give different answers-
- Understand that a problem can be solved using a variety of operations.
- Recognize that "cube" is a model/representation of multiplying 3 factors - 3 factors can be multiplied in any order and the products are equal-

Unit 3: MULTIPLICATION AND DIVISION FACTS

<p>Chapter 8: Practice Multiplication and Division Facts</p> <p>Lesson 1: Relate Multiplication and Division</p> <p>Lesson 2: Multiply & Divide Facts Through 5</p> <p>Lesson 3: Multiply & Divide Facts: Through 10</p> <p>Lesson 4: Multiplication Table Through 12</p> <p>Lesson 5: Multiply 3 Factors</p> <p>Lesson 6: Problem Solving Skill</p>	<p>Chapter 9: Algebra: Use Multiplication and Division Facts</p> <p>Lesson 1: Expression with Parentheses</p> <p>Lesson 2: Match Words and Expressions</p> <p>Lesson 3: Multiply Equals by Equals</p> <p>Lesson 4: Expressions with Variables</p> <p>Lesson 5: Equations with Variables</p> <p>Lesson 6: Find a Rule</p> <p>Lesson 7: Problem Solving Strategy</p>
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GRADE 4
Harcourt Mathematics

MODULE 2: Multiplication and Division Facts
3 Weeks of Instruction
Unit 3: Chapters 8-9

<p><u>Day 1:</u> Chapter 8: Practice Multiplication and Division Facts</p> <p>Lesson 8.1 Relate Multiplication and Division</p>	<p><u>Day 2:</u> Lesson 8.2 Multiply and Divide Facts Through 5</p>	<p><u>Day 3:</u> Lesson 8.3 Multiply and Divide Facts Through 10</p>	<p><u>Day 4:</u> Lesson 8.4 Hands-On: Multiplication Table Through 12</p>	<p><u>Day 5:</u> Lesson 8.5 Multiply Three Factors</p>
<p><u>Day 6:</u> Chapter 8</p> <p>Lesson 8.6 Problem Solving Skill: Choose the Operation</p>	<p><u>Day 7:</u> Chapter 9: Algebra: Use Multiplication and Division Facts</p> <p>Lesson 9.1 Expressions with Parentheses</p>	<p><u>Day 8:</u> Lesson 9.2 Match Words and Expressions</p>	<p><u>Day 9:</u> Lesson 9.3 Multiply Equals by Equals</p>	<p><u>Day 10:</u> Lesson 9.4 Expressions with Variables</p>
<p><u>Day 11:</u> Lesson 9.5: Equations with Variables</p>	<p><u>Day 12:</u> Lesson 9.6: Find A Rule</p>	<p><u>Day 13:</u> Lesson 9.7: Problem Solving Strategy: Work Backward</p>	<p><u>Day 14:</u> CA Connection Unit Review Assessment.</p>	

Day 14: CA Connection and Unit Review/Assessment: This Module does not contain a lesson for Day 14.

UNIT 3: MULTIPLICATION AND DIVISION FACTS

Module 2: Chapter 8

DAY 1: LESSON: 8.1 , pp. 140-141

Materials:	*Transparency 8.1; Color Tiles - 1 bag of 50 tiles per groups of 3 - 4 students;																		
LESSON FOCUS:	Relate Multiplication and Division Facts																		
CALIFORNIA STANDARDS:	<p>Number Sense: 3.0: Students solve problems involving addition, subtraction, multiplication and division of whole numbers and understand the relationships among operations. Mathematical Reasoning: 1.1, 2.0, 2.1, 3.0, 3.2 1.1: Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, sequencing and prioritizing information and observing patterns.</p>																		
Purpose of Lesson:	<p>Understand how to relate multiplication and division: Fact families have 4 different equations, 2 for each operation; the 2 factors in multiplication can be interchanged to form another equation- Division is the inverse of multiplication; it breaks the product apart in two different ways-the factors become the divisor and quotient-</p>																		
<p>LAUNCH: *Transparency 8.1</p> <p>Materials: Color Tiles 1 bag of 50 tiles per groups of 3 -4 students</p>	<p>Warm up: Number of the Day - discuss students' equations Ask if anyone used fractions? Division? Multiplication?</p> <p>Discuss: How are multiplication and division related? Chart student responses</p> <p>Pg. 140B: Use "Alternative Teaching Strategy & Vocabulary Strategy, Pg. 139: Discuss arrays - Problems: 15- 17 Students name multiplication and division expressions:</p> <div style="text-align: center;"> <p>5</p> <table style="margin: auto;"> <tr> <td style="border: 1px solid black; width: 20px; height: 20px;"></td> <td style="border: 1px solid black; width: 20px; height: 20px;"></td> <td style="border: 1px solid black; width: 20px; height: 20px;"></td> <td style="border: 1px solid black; width: 20px; height: 20px;"></td> <td style="border: 1px solid black; width: 20px; height: 20px;"></td> <td style="border: 1px solid black; width: 20px; height: 20px;"></td> </tr> <tr> <td style="border: 1px solid black; width: 20px; height: 20px;"></td> <td style="border: 1px solid black; width: 20px; height: 20px;"></td> <td style="border: 1px solid black; width: 20px; height: 20px; text-align: center;">15</td> <td style="border: 1px solid black; width: 20px; height: 20px;"></td> <td style="border: 1px solid black; width: 20px; height: 20px;"></td> <td style="border: 1px solid black; width: 20px; height: 20px;"></td> </tr> <tr> <td style="border: 1px solid black; width: 20px; height: 20px;"></td> <td style="border: 1px solid black; width: 20px; height: 20px;"></td> <td style="border: 1px solid black; width: 20px; height: 20px;"></td> <td style="border: 1px solid black; width: 20px; height: 20px;"></td> <td style="border: 1px solid black; width: 20px; height: 20px;"></td> <td style="border: 1px solid black; width: 20px; height: 20px;"></td> </tr> </table> </div> <p style="margin-left: 100px;">3</p> <p style="margin-left: 500px;">Factors: 3 & 5 Product: 15</p> <p style="margin-left: 100px;"> $3 \times 5 = 15$ $15 \div 3 = 5$ $5 \times 3 = 15$ $15 \div 5 = 3$ </p> <p>Introduce the notation: $3 \times 6 = N$ Discuss the meaning of "N" in this equation - generate others with students including: $2 \times N = 12$ - What is "N" in this case?</p>									15									
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EXPLORE: Work with the concept. Focus on student's "doing mathematics."	<p>Question: If there are 16 students in the band, show all the different ways they can line-up with same number of students in each line: Students work with partners and use color tiles to find all the possible arrangements, and sketch and label arrangements.</p>																		

	Discuss results, Record the multiplication and division "facts"
PRACTICE: Focus on Communication and Representation	Pg. 141: Problems 2-5: Do with students: Represent the solution and the multiplication and division fact. Pg. 140: Use term "inverse operation" Pg. 141: #s: 18 - 20: Partner work: Students show representation and facts.
SUMMARIZE: Connect purpose to activities	Pg. 141: Assess: "Discuss" - When given the equation $4 \times 3 = 12$, describe how you would find the other equations in the fact family.
CLOSURE: Assess Individually/group	Pg. 141: Assess: "Write" - Describe the relationship between multiplication and division. Use the vocabulary: inverse operations Chart student responses
HOMEWORK:	Pg. 141: Problems 19; Write a word problem for $6 \times 7 = 42$ and 42 divided by $6 = 7$

ROUTINE:

Today's Number:

Introduce Routine: See attached description of this routine.
Students work individually/with a partner.

Discuss students' strategies for developing number sentences/representations for "Today's Number".

Encourage equations that use more than 1 operation.

Suggestions for 4th Grade -Today's Numbers:

10, 20, 24, 25, 36, 50, 75, 100

This routine develops' students ability to "decompose" numbers so they can more easily operate with numbers if they have strategies for working with equivalent components of numbers.

Ex: $24 = 10 + 10 + 4$ 6×4 $1/2$ of 48

Routine: Today's Number

Today's Number is a routine that gives students an opportunity to practice and extend understanding of familiar and developing number concepts. Students need a journal to use everyday that is separate from their math journal. Give them a chance to decorate the journal with the title and their name. As students integrate new concepts into the routine, they will add new words and symbols to the cover of the journal. You can begin Today's Number with number one on the first day of school and continue counting days throughout the year, or you can use another sequence of numbers. The important thing is that students can apply the new ideas they learn one day, to the work they do the next day.

The daily routine should extend no longer than 20 minutes. The first ten minutes, students are silently brainstorming equations that equal Today's Number on their own. As students begin their own work each day, the teacher can begin constructing the class chart which will be used for sharing at the end of the session. As the teacher walks around the room, observing student thinking, she/he can look for students who are pushing themselves and experimenting with different methods of expressing Today's Number. Mini-conferences are encouraged as they provide a chance to teach students individually. When students come up with new or interesting ways of representing the number, the teacher can add the idea to the class chart during the brainstorm. This gives the teacher a chance to honor individual thinking and also encourages other students to consider new ideas.

After ten minutes, students share with a partner or with their table group, adding their favorites from their partner or group to their own work. The last 6-8 minutes are spent as a whole class, the teacher recording student's ideas on the class chart. As the teacher opens up the share, he/she can talk to the class about the equations that already appear on the chart. This gives the teacher an opportunity to clarify misconceptions about rules and procedures (parentheses, order of operations, numerical notation, etc.) The share also provides a useful record of the collective knowledge of the class and gives the teacher information about what new concepts students are ready to start thinking about. Today's Number also serves as an ongoing assessment of students' individual progress. After the year has passed students are proud to see how much they have learned about number concepts and mathematics.

Suggestions for introducing Today's Number: Each week, increase the level of thinking required for Today's Number. Maintain a class chart, so that students can keep track of weekly additions. This list is only a suggestion. You can change the order if necessary to make sure new concepts correlate with instruction. The italicized words and corresponding symbols can be added to the journal cover as they are introduced.

Week 1: Students practice brainstorming equations that equal Today's Number. Clarify the definition of the word, *equation*.

Week 2: Encourage students to include at least two *subtraction* equations if they aren't already doing so.

UNIT 3: MULTIPLICATION AND DIVISION FACTS

Module 2: Chapter 8

DAY 2: LESSON: 8.2, pg. 142-143

MATERIALS:	*Transparency 8.2; Color Tiles: 1 bag of 50 per 3-4 students; Transparency of Challenge 8.2
LESSON FOCUS:	Multiplication and Division Facts Through 5
CALIFORNIA STANDARDS:	Number Sense: 3.0: Students solve problems involving addition, subtraction, multiplication and division of whole numbers and understand the relationships among operations. Mathematical Reasoning: 2.0, 2.3, 3.0, 3.2, 3.3
Purpose of Lesson:	Understand how to multiply and divide using facts from 0 through 5: Connect skip-counting models to fact families and inverse operations. Multiplication chart to find "a X b" and "c divided by d" See Pg. 142A: Math Background
LAUNCH: *Transparency 8.2 Color Tiles: 1 bag of 50 per groups of 3-4 students	*Warm-up: Number of the Day: Students explain how they found all the possible equations. HW: Students share problems with partner Pg. 142 B: Use Alternative Teaching Strategy: Students use color tiles to model equations: $4 \times 3 = N$ $3 \times 4 = N$ N divided by 3 = _____ N divided by 4 = _____ Students should show 3 groups of 4 or 4 groups of 3 for Each equation. Question: What are all the different "models" you can use to show multiplication and division? Record student responses. Add those shown on Pg. 142 (Learn) that students did not mention. Label each of the models: "groups of"; number line or skip counting; arrays-
EXPLORE: Work with the concept. Focus on student's "doing mathematics."	Pg. 143: Partner work: Problems 2-7; Discuss students' responses. Ask for explanations about how they found the "related" fact and justify their response with a model. Pg. 143: Problems 18 - 19: Solve and discuss with students.
PRACTICE: Focus on Communication and Representation	Pg. 143: Problems 8 - 13: Solve and write at least 1 story problem for one of the equations, include a "model/diagram" as part of the answer. Students share problems and match equation to story problem.

Overhead: Challenge 8.2	Challenge 8.2: Solve problems and discuss strategies.
<p>SUMMARIZE/ CLOSURE: Connect purpose to activities</p> <p>Assess Individually</p>	<p>Pg. 143: Assess: Discuss: Students describe their strategy (ies) for solving the problem.</p> <p>Student discuss/write how the different representations (On Pg. 142) from Emma, Blake, Carlos, and Latoya and the multiplication chart - connect multiplication to division.</p> <p>Ask students to identify which is the most useful/makes the most "sense" to them.</p>
HOMEWORK:	<p>Pg. 143: Problems 20 - 24.</p> <p>Students provide models/representations for Problems 22 - 24.</p>

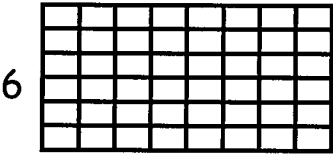
ROUTINES:

Continue with Routines from DAY 1.

UNIT 3: MULTIPLICATION AND DIVISION FACTS

Module 2: Chapter 8

DAY 3: LESSON: 8.3, pg. 144 - 145

MATERIALS:	*Transparency 8.3; CM Grid paper - TR 60 1 per students or color tiles
LESSON FOCUS:	To multiply and divide facts from 6 - 10
CALIFORNIA STANDARDS:	<p>Number Sense: 3.0: Students solve problems involving addition, subtraction, multiplication and division of whole numbers and understand the relationships among operations. 4.0: Students know how to factor small whole numbers. Mathematical Reasoning: 1.2 and 2.3</p>
Purpose of Lesson:	<p>Understand/learn multiplication and division facts from 6 - 10: Recognize that multiplication and division are inverse operations, every multiplication fact in the table has a corresponding division fact. Recognize and use the doubling strategy in multiplication as an approach to multiplication as repeated addition. Harcourt Math calls the Commutative Property the "Order Property" - students should learn the term: Commutative Property</p>
<p>LAUNCH: *Transparency 8.3</p> <p>CM Grid paper - TR 60 1 per students or color tiles</p>	<p>*Warm-up: Problem of the Day Students to describe their approach for solving this problem.</p> <p>HW problems: Partner discussion. Share models/diagrams.</p> <p>Show a 6X8 array on board/overhead. (Optional: Students use color tiles rather than grid paper.)</p> <p style="text-align: center;">8</p> <div style="text-align: center;">  </div> <p>Question: Can you find a way to break this grid into 2 pieces with a straight line? Describe the 2 new arrays. Ex: 2 X 6 and 6 X 6 - If you figure/find the product for the 2 smaller arrays and add them together - will this equal to 6 X 8? Model the process for recording.</p>
EXPLORE:	<p>Have students try this and then justify their explanation. Ask: If you break this array into 2 small arrays "that you know" (4 X 6) or (3 X 8), how does this help you find the product of 6 X 8? Ask students to "explain" their response. Try this with another array such as 6 X 7 or 6 X 9.</p>

	<p>Discuss strategies with students</p> <p>Ask students for other strategies that they use for learning multiplication and division facts.</p> <p>See Pg. 145 for examples:</p> <p>Record students' response and label, give example according to the "type":</p> <ul style="list-style-type: none"> Inverse- Commutative Property- Using a pattern- Break apart-
PRACTICE:	<p>Pg. 146, Problems: 7 - 8: Students sketch and label each of the arrays.</p> <p>Pg. 146 and 147: Partner work: Problems: 31 - 33</p> <p>Discuss problems and approaches with students.</p>
SUMMARIZE/ CLOSURE:	<p>Pg. 147: Problems 28 - 29. Chart students' response</p> <p>Pg. 147: Assess: "Discuss"</p> <p>Pg. 146: Discuss: Problem 27</p>
HOMEWORK:	<p>Pg. 147: Link-up to Health, Problems 1- 3 and 40 - 41</p>

ROUTINE:

T.E. PG. 138 G: Teach: Practice Game: "Cross It Out!"

UNIT 3: MULTIPLICATION AND DIVISION FACTS

Module 2: Chapter 8

DAY 4: LESSON: 8.4, pp. 148 - 149

MATERIALS:	* Transparency 8.4; Blank Multiplication Chart - 1 per student
LESSON FOCUS:	Multiplication Table through 12
CALIFORNIA STANDARDS:	<p>Number Sense:</p> <p>*3.0: Students solve problems involving addition, subtraction, multiplication and division of whole numbers and understand the relationships among operations.</p> <p>4.0: Students know how to factor small whole numbers.</p> <p>Mathematical Reasoning: 1.2, 2.0 and 2.3</p>
Purpose of Lesson:	Write multiplication facts through 12. Understand and use strategies to find products and quotients mentally.
<p>LAUNCH:</p> <p>*Transparency 8.4</p> <p>Blank Multiplication Chart - 1 per student</p>	<p>*Warm up: Problem of the day - partner check</p> <p>Do skip counting patterns: By 10s to 120 By 11s to 132 By 12s to 144</p> <p>Students record patterns and discuss "what they notice" about them.</p> <p>Investigation: Multiplication Table: What do you know about it? How is it used? How is it useful to you? Students complete the blank table with the "facts" that they already know.</p> <p>Ex: 1s, 2s, 5s, 10s, doubles - Student draw a line through facts they know.</p> <p>Complete the remainder of the chart. Discuss the "facts" that are difficult.</p> <p>Highlight: Commutative Property. Students circle the products that are difficult for them or that they do not "know". Discuss strategies for learning facts and make a list (chart) of them:</p> <ul style="list-style-type: none"> - Use an easier fact and "add on" - Skip counting - For 9s, multiply by 10 and then subtract - Break apart <p>Pg. 149: Discuss the connection to division - "Connect"</p>
EXPLORE:	<p>Partner work: Try a strategy from the class chart to "practice" 2 different facts that they do not know.</p> <p>Students explain how the strategy was helpful or not.</p>

PRACTICE:	Pg. 149: Problems: 16 -23. Confer with small groups of students, ask them to model/represent their thinking and explain their strategy for solving the problems.
SUMMARIZE/ CLOSURE:	Discuss students' approaches and solutions to Problems 22 and 23. Ask students to try other's approach to solving these problems. Pg. 149: Assess: Discuss how the break-apart strategy would work for Mrs. Richards' problem.
HOMEWORK:	Students record the multiplication facts that are difficult for them. Make a plan for "learning" all difficult multiplication facts. Describe how you will practice. Select a "target" date for learning all the facts. List the ways that the teacher/classmates can help them learn the facts.

ROUTINES:

Continue with Routines from previous lessons.