



San Diego Unified School District

Instructional Module to Enhance the Teaching of

H A R C O U R T

Math

California Edition

Grade 4

Module 6 – Revised

Unit 4: Multiply by 1- and 2-Digit Numbers

— WORK IN PROGRESS —

San Diego City Schools
Instruction and Curriculum Division
MATHEMATICS CURRICULUM MAP – GRADE 4

MODULE 6 – MULTIPLY BY 1 AND 2-DIGIT NUMBERS

Modules represent individual units of study that lead to the essential learnings

THREADS THROUGHOUT THE YEAR:

The threads represent ongoing learning opportunities in which students should be actively engaged throughout all units of inquiry during the entire school year. These items should not be isolated to any one particular unit of inquiry.

Students will:

- Develop understanding of numbers and the number system and use their understanding to solve problems and recognize reasonable results.
- Develop understanding of and fluency in basic computation and procedural skills.
- Use mathematical reasoning to solve problems.
- Communicate their mathematical thinking by using words, numbers, symbols, graphs and charts and translate between different representations.
- Use equations and variables to express generalizations of patterns and relationships.
- Develop logical thinking to analyze evidence and build arguments to support or refute a hypothesis.
- Make connections among mathematical ideas and between other disciplines.
- Develop and use strategies, skills, and concepts to solve problems.
- Use appropriate tools, including technology, as vehicles to learn mathematical concepts.

These are essential learnings that represent bigger ideas/concepts:

- *Students use the meaning of multiplication and the distributive property along with their understanding of different models used for 1-digit multiplication to multiply by 2-digit numbers.*
- *Students estimate products for reasonableness by breaking apart and rounding factors into easier to handle numbers that they can compute mentally.*

These are essential questions that learners ask themselves in order to achieve the essential learnings:

- How can I use patterns to develop my ability to multiply mentally?
- How can I use what I know about the **meaning of multiplication and the distributive property* to help me understand strategies for 2-digit multiplication?
- What are the similarities and differences between the following multiplication models: the manipulative model with base 10 pieces, the area model, the partial products algorithm and the standard algorithm?
- How can I use estimation to determine the reasonableness of answers to 2-digit multiplication problems?
- How can I use my understanding of multiplying by 1-digit numbers to multiply by 2-digit numbers?

***Presented in previous grade(s)**

Resources: Van de Walle: Chapters 10 & 13 (pp. 143-150 & 214-216), *Mathematics Source Book: Multiplication* (pp.37-47)

UNIT 4: Multiply by 1- and 2-Digit Numbers
MODULE 6 – 4 Weeks of Instruction

Key Mathematical Concepts:

- Use strategies/models such as multiplication patterns and partial products to visualize and understand the process of multiplying by 2-digit numbers.
- Know when a rounded solution is necessary/useful and explain why such a solution may be appropriate.
- Solve problems involving addition, subtraction, multiplication, and division of whole numbers using efficient and accurate strategies including the standard algorithms; understand and use the relationship between operations to solve computation problems.
- Understand how to use and interpret variables, math symbols, and properties to write and simplify expressions and sentences (equations).
- Know how to make decisions/select approaches to problems, such as analyze problems by identifying relationships or determine when/how to break a problem into simpler parts.
- Use strategies, skills, and concepts to find solutions; use estimation to verify reasonableness of results; apply strategies and results from simpler problems; use variety of methods to explain mathematical reasoning.

<p>Chapter 10: <u>Multiply by 1-Digit Numbers</u></p> <p>Lesson 1: Mental Math: Multiplication Patterns</p> <p>Lesson 2: Estimate Products</p> <p>Lesson 3: Hands On: Model Multiplication</p> <p>Lesson 4: Multiply 3-Digit Numbers</p> <p>Lesson 5: Multiply 4-Digit Numbers</p> <p>Lesson 6: Problem Solving Strategy: Write an Equation</p>	<p>Chapter 11: <u>Understand Multiplication</u></p> <p>Lesson 1: Mental Math: Patterns with Multiples</p> <p>Lesson 2: Multiply by Multiples of 10</p> <p>Lesson 3: Estimate Products</p> <p>Lesson 4: Hands On: Model Multiplication</p> <p>Lesson 5: Problem Solving Strategy: Solve a Simpler Problem</p>
<p>Chapter 12: <u>Multiply by 2-Digit Numbers</u></p> <p>Lesson 1: Multiply by 2-Digit Numbers</p> <p>Lesson 2: More about Multiplying by 2-Digit Numbers</p> <p>Lesson 3: Multiply Greater Numbers</p> <p>Lesson 4: Practice Multiplication</p> <p>Lesson 5: Problem Solving Skill: Multi-step Problems</p>	

Harcourt Mathematics

Grade 4

Unit 4: Multiplying by 1- and 2-Digit Numbers
Chapters 10 - 12**MODULE 6 NOTES****Chapter 10 Recommendations:**

- Teach Lesson 10.3: Hands-on: Model Multiplication as the Day 1 lesson rather than on Day 3.
- Teach Lesson 10.1 as Day 2 lesson – use strategies from Lesson 10.2: Estimate Products – and eliminate Lesson 10.2 as a formal lesson.

Chapter 11 Recommendations:

- Suggested re-ordering of lessons so students use the “hands-on” experiences to build understanding of multiplication.

Teach:

- Lesson 11.4: Hands-on: Model Multiplication on Day 7
- Lesson 11.1: Mental Math: Patterns With Multiplication on Day 8
- Lesson 11.5: Solve a Simpler Problem on Day 9
- Lesson 11.2: Multiply by Multiples of 10 on Day 10

The estimation concepts in Lessons 10.2 and 11.3 can be embedded in all lessons and these 2 lessons can be eliminated if additional instructional time is needed for other lessons.

This module does not have lessons for:

Day 18: California Connections and Unit Review

Day 19: Unit Review and Assessment

MULTIPLY BY 1- AND 2-DIGIT NUMBERS
4 Weeks of Instruction – Unit 4

<p><u>Day 1</u> CHAPTER 10.1: Multiply by 1-Digit Numbers Lesson 10.1 Mental Math: Multiplication Patterns</p>	<p><u>Day 2</u> *Lesson 10.2 Estimate Products</p>	<p><u>Day 3</u> Lesson 10.3 Hands-On Model Multiplication</p>	<p><u>Day 4</u> Lesson 10.4 Multiply 3-Digit Numbers</p>	<p><u>Day 5</u> Lesson 10.5 Multiply 4-Digit Numbers</p>
<p><u>Day 6</u> Lesson 10.6 Problem Solving Strategy: Write an Equation</p>	<p><u>Day 7</u> CHAPTER 11: Understand Multiplication” Lesson 11.1 Mental Math: Patterns with Multiples</p>	<p><u>Day 8</u> Lesson 11.2 Multiply by Multiples of 10</p>	<p><u>Day 9</u> *Lesson 11.3 Estimate Products</p>	<p><u>Day 10</u> Lesson 11.4 Hands On: Model Multiplication</p>
<p><u>Day 11</u> Lesson 11.5 Problem Solving Strategy: Solve a Simple Problem</p>	<p><u>Day 12</u> CHAPTER 12 Multiply by 2-Digit Numbers Lesson 12.1 Multiply by 2-Digit Numbers (Day 1 of a 2-Day Lesson)</p>	<p><u>Day 13</u> Lesson 12.1 Multiply by 2-Digit Numbers (Day 2 of a 2-Day Lesson)</p>	<p><u>Day 14</u> Lesson 12.2 More About Multiplying 2-Digit Numbers</p>	<p><u>Day 15</u> Lesson 12.3 Multiply Greater Numbers</p>
<p><u>Day 16</u> Lesson 12.4 Practice Multiplication</p>	<p><u>Day 17</u> Lesson 12.5 Problem Solving Skill: Multi-Step Problems</p>	<p><u>Day 18</u> California Connections Unit Review</p>	<p><u>Day 19</u> Unit Review and Assessment</p>	<p><u>Day 20</u> Begin Module 7</p>

DAY: 1

Unit 4: Multiply by 1-and 2-Digit Numbers

Module 6: Chapter 10: Multiply by 1-Digit Numbers

LESSON 10.1, pp. 186-187

MATERIALS:	Highlighters; crayons; markers; poster board
LESSON FOCUS:	Mental Math: Multiplication Patterns
CALIFORNIA STANDARDS:	Mathematical Reasoning 2.3 1.1: Analyze problems by identifying relationships...observing patterns... Number Sense 3.1 3.0: Solve problems involving addition, subtraction, multiplication, and division of whole numbers using standard algorithms; understand and use the relationship among operations.
PURPOSE OF LESSON:	<ul style="list-style-type: none"> Understand how to use basic multiplication facts and patterns to develop students' ability to multiply mentally. Use patterns such as the number of zeros, and using products of ten, to compute products when multiplying with large numbers without using paper and pencil.
LAUNCH: Introduce students to concepts.	Special Needs and Early Finishers (for all students) TE P. 186B. <ul style="list-style-type: none"> Discuss the patterns and how they help students determine the next product. Continue strategy: Students create similar problems using basic facts and multiplication patterns with partner.
EXPLORE: Work with the concept. Focus on students "doing mathematics."	Learn, Pg. 186: Found Money. <ul style="list-style-type: none"> Write problem on board/overhead. Students work with partner to solve using: representations and numbers. Discuss strategies and solution(s). Teach, Pg. 186: Guided Instruction questions to guide discussion. Examples A and B: Pg. 186: Create context for the number sentences (Ex: 7 plates with 20 cookies). <ul style="list-style-type: none"> Highlight and discuss/record useful patterns students recognize. Discuss Algebraic Thinking, margin TE p. 186.
PRACTICE: Focus on Communication and Representation.	Check, Pg. 186, #1: Discuss. <ul style="list-style-type: none"> Then, make chart with students the multiples for #2 – 5. Discuss patterns. Highlight basic facts and underline zeros as in Special Needs, p. 186B. Practice & Problem Solving, Pg. 187, #22 – 27. <ul style="list-style-type: none"> Discuss. Then, #28 – 32. Discuss strategies.
SUMMARIZE: Connect purpose to activities.	<ul style="list-style-type: none"> Show a sequence of products: Ex: 240; 2,400; 24,000, etc. Students identify the factors for each: (240: 4, 6, and 10; 2,400: 4, 6 and 100; 24,000: 4, 6 and 1,000). ASSESS, TE Pg. 187: WRITE. Share responses.
HOMEWORK:	Practice & Problem Solving, p. 187: # 18-21; Mixed Review and Test Prep: #s: 33 -37.

ROUTINES:**Find the Factors**

Write the following one at a time.

Students give all possible factors for each, omitting 1 and the number itself,
e.g., 12: 2, 3, 4, 6

18 15 30 42 24 36 45 27 14

Students write numbers having three or more factors (omitting 1 and the number itself), and list the factors, e.g., 20: 2, 4, 5, 10. Encourage students to include larger numbers, e.g., 48, 72, 128...

Discussion Question:

How can you be sure you have found all the factors for a number?

Discuss. Give an example of factors beginning with 2, then 3, combined with pairing the factors as a means of checking. Example:

20 is divisible by 2: $20 \div 2 = 10$, so 2 and 10 are factors of 20.

20 is not divisible by 3.

20 is divisible by 4: $20 \div 4 = 5$; so 4 and 5 are factors.

20 is not divisible by 6, 7, 8, 9, etc.

Ask if there are any other factors of 20 besides 1 and 20.

Discuss other strategies students use to determine the all the factors as numbers such as arrays and "facts".

DAY: 2
 HM Unit 4: Multiply by 1- and 2-Digit Numbers
 Module 6: Chapter 10: Multiply by 1-Digit Numbers
 LESSON 10.2, pp. 188-189

MATERIALS:	Special Needs: index cards
LESSON FOCUS:	Estimate Products
CALIFORNIA STANDARDS:	<p>Number Sense 1.4: Decide when a rounded solution is called for and explain why such a solution may be appropriate. 3.0: Solve problems involving addition, subtraction, multiplication, and division of whole numbers using standard algorithms; understand and use the relationship among operations. Mathematical Reasoning: 2.5</p>
PURPOSE OF LESSON:	<ul style="list-style-type: none"> • Understand how to estimate products by rounding to the greatest place value. • Learn how estimation helps determine the reasonableness of the answer.
LAUNCH: Introduce students to concepts	<p>Special Needs (for all students), TE Pg. 188 B.</p> <ul style="list-style-type: none"> • Students discuss strategies for solving. Ask students to place their cards on a number line. • Continue strategy by having students create and share similar problems with partners.
EXPLORE: Work with the concept. Focus on students “doing mathematics.” Students have books closed.	<p>Learn, Pg. 188: Flour Power. Write problem on board/overhead.</p> <ul style="list-style-type: none"> • Teach, p. 188: Guided Instruction questions to guide discussion of Steps and Examples A & B. • Record and discuss strategies for solving. <p>Check, Pg. 188: Do #s: 1 - 3 with students.</p> <ul style="list-style-type: none"> • Create context/situations for Problems 2-3. • Discuss rounding strategies. <p>Practice & Problem Solving, Pg. Pg. 189: #26 - 29:</p> <ul style="list-style-type: none"> • Put problems and factors on board/overhead. Students work with partners to solve. Discuss solutions and strategies.
PRACTICE: Focus on Communication and Representation.	<p>Practice & Problem Solving, Pg. 189: #30 – 34; 6-9</p> <ul style="list-style-type: none"> • For problems 30 -33: Students show/explain/justify answers with pictures and number sentences. • Students work with partners/individually. Confer as students' work. Discuss and share solutions and strategies for solving. • Ask students to round and estimate the product of 3×27. Share estimates when rounding 9 to 10 ($10 \times 27 = 270$) and 27 to 30 ($9 \times 30 = 270$). • Repeat procedure for 87×6: ($90 \times 6 = 540$) ($87 \times 10 = 870$) • Emphasize both are reasonable estimates. • Discuss which rounded answer is closer to actual product and why.

SUMMARIZE: Connect purpose to activities.	ASSESS, TE Pg. 189: DISCUSS. ASSESS, TE Pg. 189: WRITE. • Students make up their own similar problems to round and multiply and share.
HOMEWORK:	Practice & Problem Solving, Pg. 189: #s: 18 - 20 and Mixed Review and Test Prep: # 35 -39.

ROUTINES:**Mental Math: Round and Round**

Begin going around class, saying 2-digit numbers. Students round to nearest ten. Continue, saying 3-digit numbers. Students round to nearest 100. Continue with 4-digit numbers, rounding to nearest thousand.

Practice rounding and multiplying mentally by writing and saying the following. Students round only the first number; e.g., 28×6 becomes 30×6 or 180. Go around class, alternating by having one student give rounded form (30×6) and next student gives product (180).

$$\begin{array}{cccccc} 39 \times 5 & & 72 \times 3 & & 63 \times 4 & & 58 \times 2 & & 43 \times 9 \\ 15 \times 6 & & 83 \times 7 & & 24 \times 9 & & 9 \times 20 & & 91 \times 3 \end{array}$$

Continue as needed.

Reteach/reinforce any skills and concepts identified in homework check as needing attention.

DAY: 3
 HM Unit 4: Multiply by 1- and 2-Digit Numbers
 Module 6: Chapter 10: Multiply by 1-Digit Numbers
 LESSON 10.3, pp. 190-191

MATERIALS:	Base-ten blocks; Routines: 1-cm grid paper for each student (TR60); ten 10-cm 1-inch wide strips (5 each of 2 colors); masking tape for each student/group.								
LESSON FOCUS:	Hands-On: Model Multiplication								
CALIFORNIA STANDARDS:	Number Sense: 3.0: Solve problems involving addition, subtraction, multiplication, and division of whole numbers using standard algorithms, understand and use the relationship among the operations. Mathematical Reasoning: 2.3								
PURPOSE OF LESSON:	<ul style="list-style-type: none"> Understand that multiplication involves combining equal groups. Use manipulative models to facilitate visualizing the multiplication process. Understand how using Base-10 blocks connects to the process of regrouping in the multiplication algorithms. 								
LAUNCH: Introduce students to concepts Base-10 blocks	<ul style="list-style-type: none"> Quick Review, p. 190. Review orally with students. Discuss strategies. Ask students estimate the product of 12×5: Explain/discuss strategies for multiplying 12×5. Ask students to rewrite problem separating the tens and ones. $(10 + 2)$ Remind students the problem asks for 5 groups of 12. This can be written $5(10 + 2)$ or $5(10) + 5(2)$ (this is an example of the distributive property – see lesson 10.4, top p. 193). <p>Write:</p> <table style="margin-left: 20px;"> <tr> <td style="padding-right: 20px;">12</td> <td style="padding-right: 20px;">$(10 + 2)$</td> </tr> <tr> <td>$\underline{\times 5}$</td> <td>$\underline{\times 5}$</td> </tr> </table> <ul style="list-style-type: none"> Use Base 10 blocks to model: Ask students to show the product of 5 times 2 ones. Model with blocks. $(10 + 2)$ Ask for the product of 5 times 1 ten: $\underline{\times 5}$ Model with blocks. 10 <table style="margin-left: 20px;"> <tr> <td style="padding-right: 20px;">10</td> <td style="padding-right: 20px;">$\underline{\times 5}$</td> </tr> <tr> <td>$\underline{+ 50}$</td> <td></td> </tr> </table> <ul style="list-style-type: none"> Explain: This shows two <u>partial products</u> and the sum of both = complete product. So, show adding $50 + 10 = 60$. Discuss/model each step again with Base-10 blocks – connect blocks to numbers in equation. Discuss product of 60 – is it close to the <u>estimate</u> of 50? Students do 2 – 3 problems with blocks, such as 4×14; 3×17; 3×13 with students. 	12	$(10 + 2)$	$\underline{\times 5}$	$\underline{\times 5}$	10	$\underline{\times 5}$	$\underline{+ 50}$	
12	$(10 + 2)$								
$\underline{\times 5}$	$\underline{\times 5}$								
10	$\underline{\times 5}$								
$\underline{+ 50}$									

	<ul style="list-style-type: none"> • Present this problem: • Ask students to estimate the product. • Follow the same procedure of renaming 345 to hundreds, tens, ones. (300 + 40 + 5) Model with blocks. Remind students the problem asks for 2 groups of 345. • Remind students to add to find the sum of the “partial products.” Discuss process with students: <i>What numbers are being multiplied for each partial product?</i> • Repeat with other examples. • Continue as needed. <p style="text-align: right;"> $\begin{array}{r} 345 \\ \times 2 \\ \hline 690 \end{array}$ </p> <p style="text-align: right;"> $\begin{array}{r} 10 \\ 80 \\ + 600 \\ \hline 690 \end{array}$ </p> <p style="text-align: right;">} Partial Products</p> <p>See Try It, Pg. 190. Students may check problems with repeated addition. (345 + 345)</p>
<p>EXPLORE: Work with the concept. Focus on students “doing mathematics.”</p>	<p>Connect, top SE Pg. 191: Students do with partner. Discuss and record partial products and find sum. Verify the partial product by using repeated addition: Ex: $138 + 138 + 138 = \underline{\hspace{2cm}}$</p> <p>Pg. 191, #5 – 8. See Modifying Instruction, margin TE p. 190. Students work with partners/individually - solve using blocks, record partial products, and add to find product.</p>
<p>SUMMARIZE: Connect purpose to activities.</p>	<p>ASSESS, TE Pg. 191: WRITE.</p>
<p>HOMEWORK:</p>	<p>Mixed Review and Test Prep, Pg. 191</p> <ul style="list-style-type: none"> • Write and solve a word problem in which you multiply a 2- or 3-digit number by a 1-digit number. Ex: <i>There are 125 stamps on a page. How many stamps are on 3 pages?</i> Show 125×3 on 1 – 3 pages or $125 + 125 + 125 = \underline{\hspace{2cm}}$

ROUTINES:**How Many of These? How Many of Those?**
(Equivalent Measurements)

Use measuring tools to review equivalent measurements, e.g.,

Examples: $100\text{ cm} = 1\text{ m}$

$1,000\text{ m} = 1\text{ km}$

$1,000\text{ mm} = 1\text{ m}$

Show students how to make their own measuring tapes using masking tape and ten 10-cm strips (five of one color and five of another color), which they place on masking tape, alternating the colors, to make 1 meter.

Record 10 cm marks on meter tape.

Ask how many 10-cm strips are needed to show 1 meter (10).

Ask how many millimeters are in 1 cm (10). In 100 cm (1,000). If desired, students can mark millimeters.

Give a measurement: Ex: 100 cm. Ask students for equivalent measurements, i.e., 1 m or 1,000 mm. Continue with 5 m, 10 cm, etc.

Students use measuring tapes to find objects which are of given lengths.

Ex: 5 cm, 10 cm, 20 cm, 25 cm, 50 cm, 100 cm, 150 cm, 1 m, 2m, etc.

Reteach/reinforce any skills and concepts identified in homework check as needing attention.

DAY: 4
 Unit 4: Multiply by 1-and 2-Digit Numbers
 Module 6: Chapter 10: Multiply by 1-Digit Numbers
 LESSON 10.4, pp. 192-195

MATERIALS:	Pp. 131-132, Skill 28: <i>Intervention Strategies and Activities:</i> *Transparencies or worksheets for students; Base-10 blocks for small group work.
LESSON FOCUS:	Multiply 3-Digit Numbers
CALIFORNIA STANDARDS:	Number Sense 3.0: Solve problems involving addition, subtraction, multiplication, and division of whole numbers using standard algorithms; understand and use the relationship among operations. Mathematical Reasoning: 2.1, 2.3
PURPOSE OF LESSON:	<ul style="list-style-type: none"> • Understand how using Base-10 blocks connects to the process of regrouping in the multiplication algorithms. • Connect and generalize the process of multiplying 2-digit numbers to the process of multiplying 3-digit numbers. • Understand how estimating can help determine the reasonableness of the product.
LAUNCH: Introduce students to concepts Base-10 Blocks *Transparencies	Quick Review, Pg. 192. Model with blocks. Discuss. P IS 129-132: Alternative Teaching Strategy in Intervention Strategies and Activities, Skill 28: <ul style="list-style-type: none"> • This strategy combines using base-ten blocks and partial products, which are recorded on place-value tables. Activity sheets are provided on pp. 131-132. • Students use Base-10 blocks to model process.
EXPLORE: Work with the concept. Focus on students “doing mathematics.” Student’s books are closed.	Learn, Pg. 192: Mix It Up: Write problem on board/overhead. Do with students. Teach, Pg. 192; Guided Instruction questions to discuss. <ul style="list-style-type: none"> • Place Value & Multiplication, p. 193: Another Way. • See bullets in margin TE p. 193 to guide discussion. • Discuss Reasoning & Algebraic Thinking bullet with students. Highlight/connect the relationship between the Base-10 blocks, place-value chart, and expanded form to develop understanding of the process for multiplying 3-digit numbers by a 1-digit number. <ul style="list-style-type: none"> • Continue emphasis and practice using the partial products strategy (similar to place value/expanded form).

