



San Diego Unified School District

Instructional Module to Enhance the Teaching of

H A R C O U R T

Math

California Edition

Grade 4

Module 7 – Revised

Unit 5: Divide by 1- and 2-Digit Divisors
Chapters 13, 14, and 16

— WORK IN PROGRESS —

MODULE 7 – DIVIDE BY 1 AND 2-DIGIT DIVISORS

Modules represent individual units of study that lead to the essential learnings

THREADS THROUGHOUT THE YEAR:

The threads represent ongoing learning opportunities in which students should be actively engaged throughout all units of inquiry during the entire school year. These items should not be isolated to any one particular unit of inquiry.

Students will:

- Develop understanding of numbers and the number system and use their understanding to solve problems and recognize reasonable results.
- Develop understanding of and fluency in basic computation and procedural skills.
- Use mathematical reasoning to solve problems.
- Communicate their mathematical thinking by using words, numbers, symbols, graphs and charts and translate between different representations.
- Use equations and variables to express generalizations of patterns and relationships.
- Develop logical thinking to analyze evidence and build arguments to support or refute a hypothesis.
- Make connections among mathematical ideas and between other disciplines.
- Develop and use strategies, skills, and concepts to solve problems.
- Use appropriate tools, including technology, as vehicles to learn mathematical concepts.

These are essential learnings that represent bigger ideas/concepts:

- Using a variety of methods, students use the inverse relationship of multiplication and division, basic facts, partial products, partial quotients, and multiplication number patterns to solve division problems.

These are essential questions that learners ask themselves in order to achieve the essential learnings:

- **How do I use manipulatives to represent solution paths to the two models of division problems?*
- **How do I record my thinking numerically to match my actions when using a model to solve division problems?*
- **How do I use the context to make sense of the meaning of the quotient and the remainder?*
- How do I explain and show the similarities and differences between the following division models: the manipulative model with base 10 pieces the area model, the partial quotients algorithm and the standard algorithm?
- How do I use the inverse operation of multiplication to understand and solve division problems?
- How do I relate the basic facts and patterns of powers of ten in multiplication to their use in division?
- How do I use **partial products* to understand and solve division problems using partial quotients?
- How do I use basic facts and patterns in multiplication to help solve division problems and to determine reasonableness of quotients?
- How do I use the **area model* to understand the relationship between **factors, multiples, and prime and composite numbers?*

*** Presented in previous grade(s)**

Resources: Van de Walle: Chapter 10 &13 (pp 143-152 & 220); *Mathematics Source Book: Division* (pp.48-58)

UNIT 5: Divide by 1- and 2-Digit Divisors
 MODULE 7 – 20 days of instruction

Key Mathematical Concepts:

- Solve problems involving addition, subtraction, multiplication and division of whole numbers and understand the relationships among the operations; multiplication is the inverse of division-
- Understand and use the two models of division to solve problems: sharing and grouping.
 Sharing – dividing a quantity into equal groups and the amount in each group.
 Grouping – the number of “groups of” in a given quantity or amount/repeated subtraction-
- Understand how use the context or situation to “make sense of” or determine what to do with the remainder in a division problem-
- Understand how to estimate a reasonable answer for division problems-
- Know how to solve division problems: multi-digit number divided by one-digit numbers and explain the meaning of the quotient/answer-
- Understand how to divide efficiently and accurately and have strategies for recording the process; understand the standard division algorithm-
- Understand how to analyze problems by identifying relationships, sequencing and prioritizing information and analyzing patterns-
- Understand that whole numbers are either prime or composite and have strategies to determining if a number is composite or prime-
- Know how to find the factors for whole numbers between 1 – 100.

<p>Chapter 13: <u>Understand Division</u></p> <p>Lesson 13.1: Divide with Remainders Lesson 13.2: Hands On: Model Division Lesson 13.3: Division Procedures Lesson 13.4: Problem Solving Strategy: Predict and Test Lesson 13.5: Mental Math: Division Patterns Lesson 13.6: Estimate Quotients</p>	<p>Chapter 14: <u>Divide by 1-Digit Divisors</u></p> <p>Lesson 14.1: Place the First Digit Lesson 14.2: Divide 3-Digit Numbers Lesson 14.3: Zeros in Division Lesson 14.4: Divide Greater Numbers Lesson 14.5: Problem Solving Skill: Interpret the Remainder Lesson 14.6: Find the Mean</p>
<p>Chapter 16: <u>Patterns with Factors and Multiples</u></p> <p>Lesson 16.1: Factors and Multiples Lesson 16.2: Factor Numbers Lesson 16.3: Prime and Composite Numbers Lesson 16.4: Find Prime Factors Lesson 16.5: Problem Solving Strategy: Find a Pattern</p>	<p>Chapter 15 will be taught at the end of the school year.</p>

Harcourt Mathematics

Grade 4

Unit 5: Divide by 1- and 2-Digit Divisors
Chapters 13, 14, and 16**MODULE 7 NOTES****Chapter 13 Recommendations:**

- Teach: Lesson 13.2 before 13.1
- Challenge 13.1 – Number Riddles
Use as on-going Challenge throughout unit. Students develop strategies for solving **and** write riddles for classmates to share.
- Lessons 13.4 and 13.6 can be eliminated if additional instructional time is needed for other lessons.

Chapter 16:

- Lessons 16.4 and 16.5 are not included in this module. These lessons do not focus on key aspects of the Fourth Grade Standards

**Chapter 15 will be taught at the end of the school year.
The Standards do not expect 4th grade students to divide by 2-digit divisors.

This module does not have lessons for Day 19 and 20.

DIVIDE BY 1- AND 2-DIGIT DIVISORS
Unit 5: 4 Weeks of Instruction

<u>Day 1</u> CHAPTER 13: <i>Understand Division</i> Lesson 13.1 Divide with Remainders	<u>Day 2</u> Lesson 13.2 Hands On: Model Division	<u>Day 3</u> Lesson 13.3 Division Procedures (Day 1 of a 2-Day Lesson)	<u>Day 4</u> Lesson 13.3 Division Procedures (Day 2 of a 2-Day lesson)	<u>Day 5</u> *Lesson 13.4 Problem Solving Strategy: Predict and Test
<u>Day 6</u> Lesson 13.5 Mental Math: Division Patterns	<u>Day 7</u> *Lesson 13.6 Estimate Quotients	<u>Day 8</u> CHAPTER 14: <i>Divide by 1-Digit Divisors</i> Lesson 14.1 Place the First Digit	<u>Day 9</u> Lesson 14.2 Divide 3-Digit Numbers	<u>Day 10</u> Lesson 14.3 Zeros in Division (Day 1 of a 2-Day Lesson)
<u>Day 11</u> Lesson 14.3 Zeros in Division (Day 2 of a 2-Day Lesson)	<u>Day 12</u> Lesson 14.4 Divide Greater Numbers	<u>Day 13</u> Lesson 14.5 Problem Solving Skill: Interpret the Remainder	<u>Day 14</u> Lesson 14.6 Find the Mean	<u>Day 15</u> CHAPTER 16: <i>Patterns with Factors and Multiples</i> Lesson 16.1 Factors and Multiples
<u>Day 16</u> Lesson 16.2 Factor Numbers	<u>Day 17</u> Lesson 16.3 Prime and Composite Numbers (Day 1 of a 2-Day Lesson)	<u>Day 18</u> Lesson 16.3 Prime and Composite Numbers (Day 2 of a 2-Day Lesson)	<u>Day 19</u> Unit Review California Connection	<u>Day 20</u> Review Assessment Preview Next Unit

Note: Chapter 15 will be taught at the end of the school year.
 See Module Notes for explanation.

DAY 1
 Unit 5: Divide by 1- and 2-Digit Numbers
 Chapter 13: Understand Division
 4 Weeks of Instruction
 LESSON 13.1, pp. 242-243

MATERIALS:	Small counters: 25-30 for each student; *Transparency 13.1; Color Tiles for small group work.
LESSON FOCUS:	Divide with Remainders
CALIFORNIA STANDARDS:	Number Sense 3.0: Solve problems involving addition, subtraction, multiplication, and division of whole numbers and understand the relationships among the operations. 3.4: Solve problems involving division of multi-digit numbers by one-digit numbers. Mathematical Reasoning 3.2
PURPOSE OF LESSON:	<ul style="list-style-type: none"> • Understand that a remainder is the amount left over when a number cannot be divided evenly and that the remainder is always less than the divisor. • Learn how to model the process of division using manipulatives. Know how to use/make sense of the remainder.
LAUNCH: Introduce students to concepts. *Transparency 13.1 Color Tiles	Number of the Day, P. 242A Alternative Teaching Strategy, P. 242B <ul style="list-style-type: none"> - Students solve $25 \div 6$. Use counters for Step 1 or have students in front of class to model division. - Discuss both division models and explain solutions.
EXPLORE: Work with the concept. Focus on students "doing mathematics." Students have books closed.	Learn, P. 242: Model It. <ul style="list-style-type: none"> • Write problem on board/overhead. • Students explain/show strategies for solving. • Use counters/manipulatives/pictorial representation to justify solution. • Emphasize the meaning of the remainder in this context. Teach, P. 242: Guided Instruction questions to guide discussion. <ul style="list-style-type: none"> • Discuss: Math Idea, P. 242 Check, P. 242 #1 with students. <ul style="list-style-type: none"> • Discuss. Check, P. 243, # 2-5: Analyze each problem. <ul style="list-style-type: none"> • Determine if each problem has a remainder. • Discuss strategies students might use to decide if a problem will have a remainder. (Multiples; hundreds charts, etc.)
PRACTICE: Focus on Communication and Representation.	Practice & Problem Solving, P. 243: #30-33. <ul style="list-style-type: none"> • Students work with partners. Discuss. • Students use manipulatives/representations/multiplication to model or justify each solution. • Then, #21 & 25. Students write a situation for 1 problem to share. • Class determines which division problem fits the situation.

SUMMARIZE: Connect purpose to activities.	ASSESS, TE P. 243: DISCUSS: “Explain How to Use Counters...” • Chart/record student responses. ASSESS, P. 243: WRITE. Share responses.
HOMEWORK:	Practice & Problem Solving, P. 243: #s 22-24 and Mixed Review and Test Prep: #s 34-38.

ROUTINES:**Is There a Remainder?**

Use problems like those below.

Ask if there is a remainder and, if so, what it is.

$13 \div 2 =$

$31 \div 3 =$

$17 \div 5 =$

$24 \div 8 =$

$16 \div 3 =$

$21 \div 7 =$

$15 \div 3 =$

$22 \div 2 =$

$45 \div 9 =$

$17 \div 8 =$

$11 \div 6 =$

$50 \div 7 =$

$42 \div 6 =$

$49 \div 8 =$

$32 \div 6 =$

$53 \div 9 =$

$15 \div 6 =$

$41 \div 5 =$

Discuss/explain strategies for deciding if the problem has a remainder.

Emphasize/connect discussion to multiplication.

Reteach/reinforce any skills and concepts identified in homework check as needing attention.

DAY 2
 Unit 5: Divide by 1- and 2-Digit Numbers
 Chapter 13: Understand Division
 LESSON 13.2, pp. 244-245

MATERIALS:	Base-10 Blocks for small group work.
LESSON FOCUS:	Hands On: Model Division
CALIFORNIA STANDARDS:	<p>Number Sense 3.0 3.2: Demonstrate an understanding of, and ability to use, standard algorithms for multiplying a multi-digit number by a two-digit number and for dividing a multi-digit number by a one-digit 3.4: Solve problems involving division of multi-digit numbers by one-digit numbers. Mathematical Reasoning 2.3</p>
PURPOSE OF LESSON:	<ul style="list-style-type: none"> • Understand how to use base-ten blocks to model the division process. • Division is the process of separating into equal groups or determining the number of “groups of” in a quantity or amount. • Know how to use/make sense of the remainder.
<p>LAUNCH: Introduce students to concepts.</p> <p>Base-10 Blocks</p>	<ul style="list-style-type: none"> • HW: Partners write one context/word problem from Practice & Problem Solving, p. 243 # 20-25 to solve. • Share and discuss problems with class. <p>Explore, P. 244. Take a Look. Write problem on board/overhead. Teach, P. 244. Guided Instruction questions to guide discussion.</p> <p>Note: 2 Division Models.</p> <ul style="list-style-type: none"> • Partitive division: $48 \div 3$ can be thought of as putting 48 objects into 3 equal-size groups. <i>How many objects in each group?</i> • Measurement division: <i>How many groups of 3 can be made with 48 objects?</i> • Record division process on chart.
<p>EXPLORE: Work with the concept. Focus on students “doing mathematics.”</p> <p>Base-10 blocks for small group work.</p>	<p>Connect, top SE P. 244.</p> <ul style="list-style-type: none"> • Model as student do Steps. • Then, follow the above procedure for similar problems. • Students use blocks/representations to model each problem. • Record solutions and write answers in sentence form. (Ex. 58 cookies divided between 2 students gives each student 29 cookies, or 58 cookies put into bags of 2 makes 29 bags.) <p style="text-align: center;"> $58 \div 2 =$ $72 \div 6 =$ $115 \div 8 =$ $15 \div 7 =$ </p>
SUMMARIZE: Connect purpose to activities.	<p>Practice, P. 245: Problem 11.</p> <ul style="list-style-type: none"> • Discuss with students. • Connect answer to division model of the amount in each group. • If students suggest division model of the number of groups of 3 in 42, discuss that this is also a representation of a division model.
HOMEWORK:	<p>Practice, P. 245: # 1-4 Mixed Review and Test Prep # 12-16.</p>

ROUTINES:**Half of and Two Times**

Students find “half of” or “two times” given numbers.

Alternate between both operations as desired. Examples:

Half of 14

Two times 7

Half of 50

Two times 50

Half of 80

Two times 25

Half of 40

Two times 100

Half of 1,000

Two times 5,000

Half of 10,000

Two times 30,000





Half of 100,000

Two times 1,000,000

Students explain strategies for solving.

Continue as needed. Avoid examples with remainders for this activity.

Reteach/reinforce any skills and concepts identified in homework check as needing attention.

<p>Students have books closed.</p>	<div style="display: flex; align-items: flex-start;"> <div style="margin-right: 20px;"> $\begin{array}{r} 12 \\ 6 \overline{)74} \\ \underline{60} \\ 14 \\ \underline{12} \\ 2 \end{array}$ </div> <div>  $=$  </div> </div> <p> $\begin{array}{r} 60 \\ 14 \end{array} \rightarrow$ 1 ten and 4 ones 14 divided into 6 groups - gives each group 2 with 2 left over. </p> <div style="display: flex; align-items: center; margin-top: 10px;"> <div style="margin-right: 10px;"> $\begin{array}{r} 12 \\ 2 \end{array}$ </div> <div style="margin-right: 10px;">  </div> <div> $74 \div 6 = 12$ in each group. Remainder of 2 </div> </div> <p style="text-align: center; margin-top: 5px;">1 ten + 2 ones = 12</p> <p>Partial Quotients Model: (steps shown on right side of problem)</p> <div style="display: flex; align-items: flex-start; margin-top: 10px;"> <div style="margin-right: 20px;"> $\begin{array}{r} 6 \overline{)74} \\ \underline{60} \\ 14 \\ \underline{12} \\ 2 \end{array}$ </div> <div> <p>Partial Quotients:</p> <ul style="list-style-type: none"> 10 (groups of 6 = 6 x 10) 2 (groups of 6 = 2 x 6) 12 groups of 6 6 x 12 = 72 + remainder of 2 </div> </div> <ul style="list-style-type: none"> • Learn, P. 246: Bale of Turtles. Write problem on board/overhead. • Teach, P. 246: Guided Instruction questions as students model Steps with blocks as they are discussed.
<p>EXPLORE: Work with the concept. Focus on students "doing mathematics."</p>	<p>Divide and Check, P. 247:</p> <ul style="list-style-type: none"> • Write problem on board/overhead. • See Bullets, top margin TE p. 247 to guide discussion. • Identify 2 division problems: $95 \div 5$ and $96 \div 6$. • Discuss similarities and differences between the 2 problems. • Students work with partners to solve both problems using blocks. <ul style="list-style-type: none"> • Discuss process. Students explain/model process with blocks while teacher records. • Ask students how they might use multiplication to solve this problem. <p>Suggestion: Use simpler problems to help students make this connection. Ex: $15 \div 3 = 5$ 5 5 </p> <ul style="list-style-type: none"> • Discuss the multiplication fact connection. <div style="border: 1px dashed black; padding: 5px; width: fit-content; margin-left: auto; margin-top: 10px;"> <p>3 groups of 5 each or 5 3s in $15 = 3 + 3 + 3 + 3 + 3$</p> </div>
<p>PRACTICE: Focus on Communication and Representation.</p>	<p>Check, P. 247: #1: Discuss with students.</p> <p>Check, P. 247: #s 2, 3, 12, 13. Discuss. Highlight remainders are less than divisors and why.</p>
<p>SUMMARIZE: Connect purpose to activities.</p>	<p>ASSESS, TE P. 249: DISCUSS. Share responses.</p>
<p>HOMEWORK:</p>	<p>Practice & Problem Solving, P. 248: #s 17-18; 38-39. Students model/represent each problem with drawings.</p>

ROUTINES:**FFF: Find Fact Families**

Students work with a partner.

Give students multiplication/division fact family members, e.g., 6, 7, 42.

Students write four related facts:

$6 \times 7 = 42$

$7 \times 6 = 42$

$42 \div 7 = 6$

$42 \div 6 = 7$

$60 \times 7 = 420$

$70 \times 6 = 420$

$420 \div 70 = 6$

$420 \div 60 = 7$

First partners to correctly write all four facts give the next set of members,

e.g., 5, 7, 35.

As players improve, they can say equations instead of writing them.

Discuss strategies for solving these problems.

Reteach/reinforce any skills and concepts identified in homework check as needing attention.

DAY 4
 Unit 5: Divide by 1- and 2-Digit Numbers
 Chapter 13: Understand Division
 LESSON 13.3, pp. 246-249
 Day 2 of a 2-Day Lesson

MATERIALS:	Base-10 blocks for small group work.										
LESSON FOCUS:	Division Procedures										
CALIFORNIA STANDARDS:	<p>Number Sense 3.0 3.2: Demonstrate an understanding of, and ability to use, standard algorithms for multiplying a multi-digit number by a two-digit number and for dividing a multi-digit number by a one-digit number; use relationships between them to simplify computation and to check results. 3.4: Solve problems involving division of multi-digit numbers by one-digit numbers.</p> <p>Mathematical Reasoning 2.3</p>										
PURPOSE OF LESSON:	<ul style="list-style-type: none"> • Understand the concept and process of dividing larger numbers. • Use manipulatives to model/represent the division process. 										
<p>LAUNCH: Introduce students to concepts.</p> <p>Base-10 blocks for small groups.</p>	<p>Write: $73 \div 5$ on board/overhead to discuss strategies/models for solving division problems.</p> <ul style="list-style-type: none"> • Students explain/show/model division process. • Connect recording to models. Use blocks to model processes. <p>1. Divide 73 into 5 equal-size groups:</p> <div style="display: flex; align-items: center; justify-content: center;"> <div style="text-align: center; margin-right: 20px;"> $\boxed{10} \quad \boxed{10} \quad \boxed{10} \quad \boxed{10} \quad \boxed{10}$ 5 groups of 10 </div> <div style="text-align: center; margin-right: 20px;"> $= 5 \overline{) 73}$ </div> <div style="text-align: center; margin-right: 20px;"> $\begin{array}{r} 1 \\ 5 \overline{) 73} \\ \underline{50} \\ 23 \end{array}$ </div> <div style="margin-right: 20px;">(ten)</div> <div style="margin-right: 20px;"> \rightarrow 5 groups of 10 each. </div> <div> \rightarrow 23 left to divide among 5 groups. </div> </div> <div style="display: flex; justify-content: center; margin-top: 10px;"> <div style="margin: 0 10px;">$\boxed{10 + 4}$</div> <div style="margin: 0 10px;">$\boxed{10 + 4}$</div> <div style="margin: 0 10px;">$\boxed{10 + 4}$</div> <div style="margin: 0 10px;">$\boxed{10 + 4}$</div> <div style="margin: 0 10px;">$\boxed{10 + 4}$</div> </div> <p>2. Finding Partial Quotients When Dividing:</p> <p>About how many 6s are in 150? (at least 20) The <u>first partial</u> quotient is 20. $20 \times 6 = 120$ Record 20. Subtract 20 from 150. (30) <i>How many 6s are in 30?</i> 5 with a difference of 0: There is no remainder.</p> <div style="display: flex; align-items: center; margin-top: 10px;"> <div style="font-size: 4em; margin-right: 10px;">}</div> <div style="margin-left: 20px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 10px;">$6 \overline{) 150}$</td> <td>Partial Quotients</td> </tr> <tr> <td style="padding-right: 10px;">$\underline{-120}$</td> <td>20 (Groups of 6: $6 \times 20 = 120$)</td> </tr> <tr> <td style="padding-right: 10px;">30</td> <td></td> </tr> <tr> <td style="padding-right: 10px;">$\underline{-30}$</td> <td>+ 5 (Groups of 6: $6 \times 5 = 30$)</td> </tr> <tr> <td style="padding-right: 10px;">0</td> <td>25 (Groups of 6: $6 \times 25 = 150$)</td> </tr> </table> </div> </div>	$6 \overline{) 150}$	Partial Quotients	$\underline{-120}$	20 (Groups of 6: $6 \times 20 = 120$)	30		$\underline{-30}$	+ 5 (Groups of 6: $6 \times 5 = 30$)	0	25 (Groups of 6: $6 \times 25 = 150$)
$6 \overline{) 150}$	Partial Quotients										
$\underline{-120}$	20 (Groups of 6: $6 \times 20 = 120$)										
30											
$\underline{-30}$	+ 5 (Groups of 6: $6 \times 5 = 30$)										
0	25 (Groups of 6: $6 \times 25 = 150$)										

	<ul style="list-style-type: none"> • <u>Add</u> the partial quotients: $20 + 5 = 25$ (groups of 6) $150 \div 6 = 25$ • Review steps with students: <ol style="list-style-type: none"> 1. Find each partial quotient by beginning with benchmark (compatible) numbers and record it in column to right of problem. 2. Add partial quotients to find answer or final quotient with or without remainder. • Record remainder: r.
EXPLORE: Work with the concept. Focus on students “doing mathematics.”	<ul style="list-style-type: none"> • Students use partial quotient method to solve: $124 \div 10$ $109 \div 5$ $112 \div 7$ $118 \div 9$ • Discuss strategies and solutions. • Help students connect the process to standard algorithm and the steps for recording. (Partial products shows place value more clearly) • Encourage language that identifies the place value of the digits as they are recorded. <p>Practice & Problem Solving, pP. 248, Algebra #47 – 52 to reinforce and check algorithm. Students work with partners.</p>
PRACTICE: Focus on Communication and Representation.	<p>Practice & Problem Solving, P. 248-249: #s 53-54, 56, and 57-58:</p> <ul style="list-style-type: none"> • Do with students. • Students work with partners. Solutions include representations of division process.
SUMMARIZE: Connect purpose to activities.	<p>P. 248: #s 37-38:</p> <ul style="list-style-type: none"> • Discuss with students.
HOMEWORK:	<p>Practice & Problem Solving, P. 248-249: #s 23-28, 35-36, and Mixed Review and Test Prep, P. 248 #s 59-66.</p>

ROUTINES:

Reteach/reinforce any skills and concepts identified in homework check as needing attention.

