SD USD

San Diego Unified School District

Instructional Module to Enhance the Teaching of

HARCOURT

Math

California Edition

Grade 5

Module 3 – Modified

Algebra: Use Addition and Multiplication And Integers

—WORK IN PROGRESS—

San Diego City Schools
Instruction and Curriculum Division
MATHEMATICS CURRICULUM MAP - GRADE 5

MODULE 3 – ALGEBRA: USE ADDITION AND MULTIPLICATION; INTEGERS
Modules represent individual units of study that lead to essential learnings

THREADS THROUGHOUT THE YEAR:
The threads represent ongoing learning opportunities in which students should be actively engaged throughout all units of inquiry during the entire school year. These items should not be isolated to any one particular unit of inquiry.
Students will:
- Develop understanding of numbers and the number system and use their understanding to solve problems and recognize reasonable results.
- Develop understanding of and fluency in basic computation and procedural skills.
- Use mathematical reasoning to solve problems.
- Communicate their mathematical thinking by using words, numbers, symbols, graphs and charts and translate between different representations
- Use equations and variables to express generalizations of patterns and relationships.
- Develop logical thinking to analyze evidence and build arguments to support or refute a hypothesis.
- Make connections among mathematical ideas and between other disciplines
- Develop and use strategies, skills, and concepts to solve problems.
- Use appropriate tools, including technology, as vehicles to learn mathematical concepts.

These are essential learnings that represent bigger ideas/concepts:
- ***Students use variables and equations to express relationships between two quantities.***
- ***Students solve equations using models, mental strategies and understanding of operations and order of operations.***
- Students understand that a negative number is the opposite of a positive number in relation to zero, and opposite numbers have the same absolute value.
- Students use understanding of operations with whole numbers to add and subtract integers.
- Students model equivalent representations of integers and the operations using both a number line and two-colored counter models.

These are essential questions that learners ask themselves in order to achieve the essential learnings:
- **How do I identify, model, and use addition properties?*
- **How do I connect whole number concepts and strategies to solve decimal addition* and multiplication problems?
- **How do I use parentheses to indicate order of operations when solving equations?*
- How do I use models, mental strategies, and substitution to solve equations?
- How do I translate between equivalent representations (models, word problems, and numerical and algebraic expressions)?
- How do I use formulas to solve problems?
- How do I use a number line to find and explain opposite and absolute values of integers and to compare and order integers?
- How do I build, explain, and generalize sets of integer counters with an equivalent net value?
- How do I explain and model different contexts of addition of integers using two-colored counters and a number line?
- How do I use two-colored counters and the number line to explain and show that the sum of an integer and its opposite (zero pairs) equal zero?
- How do I use my understanding of zero pairs to model and explain subtraction of integers using two-color counters?
- How are the number line and two-color counter models similar, and how are they different?

- ***Presented in previous grades***

Resources: **Van de Walle: Chapter 22 (pp. 417-433); Ch. 24 (427-430);** ***Mathematics Source Book:*** **Algebra & Functions Foundations (pp. 79-87)**

ALGEBRA: USE ADDITION AND MULTIPLICATION: INTEGERS

Key Mathematical Concepts:

- Use and understand the meaning and connections between key vocabulary: absolute value; opposites; zero pairs; variable; numerical and algebraic expression and equation; evaluate; formula; solution.
- Identify, model and use properties of addition and multiplication.
- Solve equations using models, mental math strategies and substitution.
- Understand true equations are equivalent on both sides of the equal sign.
- Translate between equivalent numerical and algebraic expressions, word problems and models of situations.
- Use formulas derived from models to solve problems.
- Develop strategies to find opposite and absolute values of integers.
- Develop strategies for comparing and ordering integers.
- Explain and model different contexts of addition of integers using two colored counters and the number line.
- Generalize rules for finding the value of equivalent sets of positive and negative integers.
- Use and generalize different strategies to add positive and negative integers and subtract positive integers from negative integers.

Notes on Integer Lessons:

- ***Because fifth grade students have not previously learned procedures without meaning for operations with integers, these lessons are a chance to have students make conjectures and generalize without the constraints of memorized rules or procedures.***
- It is important to allow time to explore the integer lessons, preferably with others, in advance of teaching the lessons.
- Students each need 15-20 red and yellow two-sided counters to explore integers during the lessons and access to the counters during routines.
- The integer lessons have been modified and expanded to provide more opportunities for students to build understanding of the concepts, analyze patterns, and make conjectures before generalizing and using rules for adding and subtracting integers.
- The number line and counter models that are used in the book have been shown to be effective, but can easily just become rote procedure if students aren't given the opportunity to make meaning of the tools while they use the tools to make meaning of integers. The lessons are designed to help students make connections between the two models, which is highly recommended in Van de Walle (see p. 459).

Day 1: Chapter 5: Algebra: Use Addition Lesson 5.1 Expressions and Variables	**Day 2:** Lesson 5.2 Write Equations	**Day 3:** Lesson 5.3 Solve Equations	**Day 4:** Lesson 5.4 Use Addition Properties	**Day 5:** Lesson 5.5 Use a Formula
Day 6: Chapter 6: Algebra: Use Multiplication Lesson 6.1 Write and Evaluate Expressions	**Day 7:** Lesson 6.2 Write an Equation	**Day 8:** Lesson 6.3 Use Multiplication Properties	**Day 9:** (Day 1 of 2-day lesson) Lesson 6.4 Use the Distributive Property	**Day 10:** (Day 2 of 2-day lesson) Lesson 6.4 Use the Distributive Property
Day 11: Chapter 23: Algebra: Integers Lesson 23.1 Integers and Absolute Value	**Day 12:** Lesson 23.2 Compare and Order Integers	**Day 13:** Lesson 23.3 Build and Generalize Equivalent Integer Sets	**Day 14:** Lesson 23.3 Use Counter Model to Add Integers	**Day 15:** Lesson 23.3 Use Number Line and Counter Models to Add Integers
Day 16: Lesson 23.3 Build True Equations	**Day 17:** Lesson 23.4 Use Counter Model to Subtract Integers	**Day 18:** Lesson 23.5 Use Counter Model and Number Line to Subtract Integers	**Day 19:** Lesson 23. Problem Solving Strategy: Draw a Diagram	

Day 1
Use Addition: LESSON 5.1
UNIT 2: ALGEBRA, DATA, AND GRAPHING

<table>
<tr><td>LESSON FOCUS:</td><td>Expressions and Variables</td></tr>
<tr><td>CALIFORNIA STANDARDS:</td><td>AF 1.2: To write and evaluate numerical and algebraic expressions involving addition and subtraction</td></tr>
<tr><td>Purpose of Lesson/Essential Question:</td><td>• How do I translate between word problems and numerical and algebraic expressions using variables?
• How do I write and evaluate numerical and algebraic expressions involving addition and subtraction and explain the meaning of the variable?</td></tr>
<tr><td>LAUNCH:

<table><tr><th>Numerical Expression</th><th>Algebraic Expression</th><th>Numerical Equation</th><th>Algebraic Equation</th></tr><tr><td>5 + 4</td><td>5 + n</td><td></td><td></td></tr><tr><td>20-12</td><td>n-12</td><td></td><td></td></tr><tr><td>14=7</td><td>14-n</td><td></td><td></td></tr></table>
Save the chart for Day 2 when you will repeat the activity using equations.

• See “Algebraic Thinking” explanation (margin, T.E., p. 70) about the use of variables.</td><td>ELL Strategy: Algebra Terms, T.E. p. 66G.
• Display the triangular diagram on a chart to introduce algebra terms. Leave the chart up for reference.
• Focus students’ attention on the terms <u>variable</u> and <u>expression</u>.
• Use the vocabulary as you refer to the triangle and during all lessons; encourage students to use the vocabulary throughout the lessons. Scaffold by referring to visuals, using the vocabulary in context, restating, and focusing on the concepts, not the definitions of the terms.
• Special Needs, T.E. p. 68B. (Modified to include numerical expressions):
• Begin by offering a numerical <u>expression</u> followed by the related algebraic <u>expression</u> given in the book, i.e., students write the numerical expression for “five plus four” (5 + 4) followed by the algebraic expression for “five plus a number” (5 + n); Twenty decreased by twelve” (20 – 12); and “a number decreased by twelve” (n – 12); “The difference between fourteen and seven,” (14 – 7); and “the difference between fourteen and a number” (14 – n).
• Record each expression in the chart as students write the expressions.
• Write 3 algebraic expressions and ask students to translate them into words: n + 6; 11 – n; and n – 42.
• “How is a numerical expression similar to and different from an algebraic expression?” The algebraic <u>expression</u> has a <u>variable</u> and the numerical <u>expression</u> only uses numbers.</td></tr>
</table>

Function Tables or "What's my Rule?" (A.T.S., T.E. p. 70): As the unit progresses, students

<table>
<tr>
<td>EXPLORE:

Focus on:
• "Doing Mathematics";
• Communication and Representation.</td>
<td>Practice & Problem Solving, S.E. p. 70, #s 13 -16.
• Do #13 together, modeling how to translate words into mathematical symbols; record in the chart below.
Partner Work:
• Partners record the algebraic expression and the meaning of the variable for #s 14-16.

<table>
<tr><th>Situation</th><th>Algebraic Expression</th><th>What does the variable represent?</th></tr>
<tr><td>13. Don lost some toys</td><td>37 – t</td><td>t = the # of toys Don lost</td></tr>
<tr><td>14. Holly's books</td><td>b + 9</td><td>b = the # of books Holly bought</td></tr>
</table>
Write a Situation (paper may be collected):
• Students each write a situation and add the algebraic expression and the meaning of the variable to their chart.
• Read the situation to a partner or table group and discuss the variable and expression.

Practice & Problem Solving, p. 70 – 71, #s 24, 26, 29 and 31.</td>
</tr>
<tr>
<td>SUMMARIZE:
Focus conversation around the Essential Questions for this lesson.</td>
<td>• How do I translate between word problems and numerical and algebraic expressions?
• How do I write and evaluate numerical and algebraic expressions involving addition and subtraction and explain the meaning of the variable?

• Select one or two students to read aloud the situations that they wrote.
• Add the equation and the meaning of the variable to the chart.
• Share solutions for #s 26 and 31.
• T.E. LESSON QUIZ 5.1, T.E. p. 71:
• Discuss #s 1 and 3.
• Students independently complete #s 2 and 4.</td>
</tr>
<tr>
<td>HOMEWORK/ PRACTICE:</td>
<td>Practice & Problem Solving, p. 70, #s 18, 23, 26, and 28
Mixed Review, p. 71</td>
</tr>
</table>

can be "rule makers" any time throughout the day to practice using mental math to identify expressions with variables.

To practice using the terms *variable* (n) and *expression* (n + 7), the last row of the chart reads, "For any number (n) the output is the (*n* + 7)." This gives practice generalizing numerical patterns. Function Tables on T.E., p. 68B, can provide an ongoing challenge to students.

See Alternative Teaching Strategy, p. 70 and Input Output in fifth grade routines.

DAY 2
UNIT 2: Use Addition: LESSON 5.2

<table>
<tr><td>MATERIALS:</td><td></td></tr>
<tr><td>LESSON FOCUS:</td><td>Write Equations</td></tr>
<tr><td>CALIFORNIA STANDARDS:</td><td>A.F. 1.2: Use a letter to represent an unknown number; write and evaluate simple algebraic expressions in one variable by substitution.</td></tr>
<tr><td>Purpose of Lesson/ Essential Question(s):</td><td>• How do I write algebraic equations involving addition and subtraction?
• How do I translate between equations and word problems?
• How do I explain the meaning of the variable?
• How do I explain the difference between an equation and an expression?</td></tr>
<tr><td>LAUNCH:

Student Books Closed

It is helpful to post words that can be substituted for plus and minus so students can refer them.</td><td>• Special Needs, p. 68B, (modified for equations):
• Refer back to triangular diagram of Algebra Terms from Day 1. Focus students' attention on equation and variable.
<table>
<tr><th>Numerical Expression</th><th>Algebraic Expression</th><th>Numerical Equation</th><th>Algebraic Equation</th></tr>
<tr><td>5 + 4</td><td>5 + n</td><td>5+4 =9</td><td>5 + n = 9</td></tr>
<tr><td>20-12</td><td>n-12</td><td>20-12 = 8</td><td>n –12 =8</td></tr>
<tr><td>14-7</td><td>14-n</td><td>14-7=7</td><td>14-n=7</td></tr>
</table>
• Change numerical and algebraic expressions from Day 2 into numerical and algebraic equations, i.e. "five plus four is nine" (5 +4 = 9) and "five plus a number is nine" (5 + n = 9); "20 decreased by 12 is eight" (20 – 12 = 8) and "a number decreased by twelve is eight" (n-12 = 8).
• Record the equations on the chart next to the expressions from Day 2.
• Ask students to translate equations into words: 16 – n = 14; n + 7 = 40; n – 72 = 37.
• Refer to the chart: "How are equations similar to expressions?" "How are they different?"
Learn, p. 72 (modified).
• Write the equation: 2 + f = 29
• Ask students to translate the equation into words.
• Display the problem, HISSSS, about the python and ask students to explain how the equation is related to the problem. "What are the two parts of the problem that are equal in value?" "What is the meaning of the variable, f?"
• Display Sample problems A and B on p. 72. For each problem work with the students to translate their solution path for the problem into an equation with a variable. Label the meaning of each part of the equation (see p. 72).
<table>
<tr><th><u>What does the variable represent?</u></th><th><u>Equation</u></th></tr>
<tr><td>A. c = birthday cd's</td><td>12 + c = 29</td></tr>
<tr><td>B. m = money to start with</td><td>m – 7 = 5</td></tr>
</table>
</td></tr>
</table>

<table>
<tr>
<td>EXPLORE:

Focus on:
• "Doing Mathematics";
• Communication and Representation.</td>
<td>Practice & Problem Solving, p. 73, #s 5, 7, 9, & 13
• Have students work with partners and record their equations in the chart.
• Sample questions:
"What does the variable in the equation represent?"
"What is the equation that represents the situation, and how did you decide?"
<u>What does the variable represent?</u> <u>Equation</u>
d = number of mph that is the difference in their speed 70 – 50 = d
Write a Situation (paper may be collected):
• Write a situation involving a variable that can be solved using the snake bar graph on p. 73.
• Add the equation and meaning of the variable to the student chart and read it to a partner.
• Write a situation to match the equation and meaning of the variable listed in your partner's chart.
• Select 2-3 students to add their equations and variable to the class chart for use during the summary.</td>
</tr>
<tr>
<td>SUMMARIZE:
Focus conversation around the Essential Questions for this lesson.

It is helpful to assess student-written problems over time</td>
<td>• How do I write algebraic equations involving addition and subtraction?
• How do I translate between equations and word problems?
• How do I explain the meaning of the variable?
• How do I explain the difference between an equation and an expression?

• Fill in the chart for the meaning of the variable and any equivalent equations for #s 7, 9, and 13. IMPORTANT: Accept variations on the equation setups, e.g., 70 – 50 = d or 50 + d = 70.

Possible questions:
• "How is an equation different from an expression?" Refer to the charts.
• "How would you change each of the equations on the chart to expressions?"

• Ask groups to use the snake bar graph and the information in the chart from a student-written situation to create a problem that could be solved using the equation (assign one situation to more than one group).
• After discussing the situations created in the groups, have the author read the original situation.</td>
</tr>
<tr>
<td>HOMEWORK/ PRACTICE:</td>
<td>Practice & Problem Solving, p. 73 #s 4, 6, 8 & 17
Mixed Review, p. 73</td>
</tr>
</table>

DAY 3

UNIT 2: ALGEBRA, DATA, AND GRAPHING

Use Addition: Lesson 5.3

LESSON FOCUS:	**Solve Equations**
CALIFORNIA STANDARDS:	A.F. 1.2: Use a letter to represent an unknown number; write and evaluate simple algebraic expressions in one variable by substitution.
Purpose of Lesson:	• How do I write and solve equations using mental math and substitution? • How do I verify that the expressions on either side of the equal sign in the equation are equal in value?
LAUNCH: Highlight Math Idea, p. 75: *"The value of the variable that makes the equation true is the **solution** of the equation."* Use the term solution naturally in context.	**Alternative Teaching Strategy, p. 74B**. True/False Equations • Substitute *"Solution to the Equation"* for *"Variable Value"* in the middle column of the chart on p. 74B. • Refer students to the term ***solution*** in the triangle. • Uncover one row of the chart at a time. • Ask students to describe the reasoning they used to determine if the equation was true or false. • Mental math and substitution are strategies that may come up. **Learn, p. 74.** • Write "ZZZZZ" problem on the board/overhead. • Ask students to write equations that model the situation. Record the equations (22+ h = 24 or 24 – 22 = h). • Discuss finding the ***solution*** to the equation using substitution and mental math (see examples on p. 74 and 75).
EXPLORE: Focus on: • "Doing Mathematics"; • Communication and Representation.	**Partner Work:** • **P.76:** Students start with # 27 and alternate finding the solution to an equation using mental math. Stop after a short time. **Practice & Problem Solving, p. 76, #s** 54-56 and select from 38-40: • Partners verbalize solution paths before writing an equation that matches the verbal path for #s 54-56. • Select 2 or 3 students to display an equation and **solution** to #56.
SUMMARIZE: *Focus conversation around the Essential Questions for this lesson.*	• *How do I write and solve equations using mental math and substitution?* • *How do I verify that the expressions on either side of the equal sign in the equation are equal in value?* **Possible questions for discussion:** • *"Where are the variables in your solutions?"* • *"Can you identify expressions? Equations?"* • *" How do you decide if the equation represents the problem?"* • *"How can translation of situations into expressions and equations help make problem solving easier?"* • *"How can you verify that the values of the expressions on both sides of the equal sign are equal in value?"* **T.E. ASSESS, p. 77: WRITE OR LESSON QUIZ #4 and 5.**
HOMEWORK:	**Mixed Review, p. 77** **Linking up to Reading, p. 77**

DAY 4

UNIT 2: Use Addition: Lesson 5.4

<table>
<tr><td>LESSON FOCUS:</td><td>Use Addition Properties</td></tr>
<tr><td>CALIFORNIA STANDARDS:</td><td>A.F. 1.2 : Use a letter to represent an unknown number; write and evaluate simple algebraic expressions in one variable by substitution.</td></tr>
<tr><td>Purpose of Lesson:</td><td>• How do I identify and use addition properties?</td></tr>
<tr><td>LAUNCH:

Goal is for students to understand and use the properties. Encourage students to refer to charts when they need to recall the name of the property or other mathematical term.

The book uses “zero property” instead of “identity property”.</td><td>Make a three-column chart on the board (see below).
• Present the first two equations in column one.
• Ask students to think privately and share with a partner what they notice about the equations.
• Students determine the value of the variable for the third equation in column one.
• Students each write an equation and explain to a partner why the equation fits the pattern.
• Add three student equations to the column.
• “Does the equation fit the pattern in the column?” “How do you know?”
• Ask students to describe the associative property in their own words.
• “What does it mean to associate with others? How do the equations relate to the meaning of associate?”
Addition Properties
<table><tr><td>Associative Property</td><td>Commutative Property</td><td>Identity Property</td></tr><tr><td>(12 + 8)+ 3 =12 + (8 + 3)</td><td>17 + 4 = 4 + 17</td><td>6 + 0 = 6</td></tr><tr><td>3 + (5 + 4) = (3 + 5) + 4</td><td>12 + 3 = 3 + 12</td><td>5.9 + 0 = 5.9</td></tr><tr><td>(16 + 5) + 4 = 16 + (n + 4)
n =</td><td>15 + n = 3 + 15
n =</td><td>n + 1.5 = 1.5
n =</td></tr><tr><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td></tr></table></td></tr>
<tr><td>EXPLORE:

Focus on:
• “Doing Mathematics”;
• Communication and Representation.</td><td>Partner Work:
• Work together to complete the columns for the commutative and identity properties, adding equations that fit the pattern.
• Write a description of each property in words.
Practice & Problem Solving, p. 79, #s 17-22.
• Partners write an equation to represent their solution path and identify the property of addition that can be used to solve the equation (refer to the chart).
• Students explain their thinking for #22 to others in the group.
Do the associative, commutative and identity properties work for subtraction?
• Partners test the three properties with subtraction problems. Students should be prepared to justify using examples.</td></tr>
</table>

SUMMARIZE:

Focus conversation around the Essential Question for this less

Post the chart for reference (it will be used in Day 8).

Routines provide opportunities to connect properties to student strategies in mental math.

How do I identify and use addition properties?

Possible discussion points:

- Students write the algebraic equations in problems 17 – 19, p. 79, in the appropriate columns of the chart (see last row of chart below). *"How does a + b = b + a explain the associative property?"*
- *"What does it mean to commute to work? How does the meaning of commute relate to the commutative property?"*
- *"How does the name, identity property relate to the pattern of the equations in the column?"*
- Surface the idea that when the same variable appears in an expression or equation, it represents the same number.
- *"Do the associative, commutative and identity properties work for subtraction?"* Students justify their conclusions using examples for each property.
- Ask students to use addition properties to mentally solve: 96 + 11 + 4.

Addition Properties

Associative Property	Commutative Property	Identity Property
(12 + 8)+ 3 =12 + (8 + 3)	17 + 4 = 4 + 17	6 + 0 = 6
3 + (5 + 4) = (3 + 5) + 4	12 + 3 = 3 + 12	5.9 + 0 = 5.9
(16 + 5) + 4 = 16 + (n + 4)	15 + n = 3 + 15	n + 1.5 = 1.5
(a + b) + c = a + (b + c)	**a + b = b + a**	**a + 0 = a**

HOMEWORK/ PRACTICE:

Practice & Problem Solving, p. 79, #s 11, 13, 15
Mixed Review, p. 79