



San Diego Unified School District

Instructional Module to Enhance the Teaching of

HARCOURT

Math

California Edition

Grade 5

Module 7 – Reorganized

Number Theory; Fraction Concepts and
Addition and Subtraction of Fractions

— WORK IN PROGRESS —

San Diego City Schools
Instruction and Curriculum Division
MATHEMATICS CURRICULUM MAP – GRADE 5

MODULE 7 – NUMBER THEORY; FRACTION CONCEPTS; ADDITION AND SUBTRACTION OF FRACTIONS

Modules represent individual units of study that lead to essential learnings

THREADS THROUGHOUT THE YEAR:

The threads represent ongoing learning opportunities in which students should be actively engaged throughout all units of inquiry during the entire school year. These items should not be isolated to any one particular unit of inquiry.

Students will:

- Develop understanding of numbers and the number system and use their understanding to solve problems and recognize reasonable results.
- Develop understanding of and fluency in basic computation and procedural skills.
- Use mathematical reasoning to solve problems.
- Communicate their mathematical thinking by using words, numbers, symbols, graphs and charts.
- Use equations and variables to express generalizations of patterns and relationships.
- Develop logical thinking to analyze evidence and build arguments to support or refute a hypothesis.
- Make connections among mathematical ideas and between other disciplines and translate between different representations
- Develop and use strategies, skills, and concepts to solve problems.
- Use appropriate tools, including technology, as vehicles to learn mathematical concepts.

These are essential learnings that represent bigger ideas/concepts:

- Students use exponential notation to represent repeated products of the same factor, to represent powers of ten; and to represent prime factorization of numbers.
- Students estimate and compute with fractions using fraction sense and their understanding of the properties of and relationships between the operations for whole numbers.
- Students understand that equivalent fractions describe the same amount using different sized fractional parts and that the more parts used to make the whole, the smaller the parts.
- Students use benchmarks for ordering and comparing fractions and represent fractions with set models, area models, and models of length.

These are essential questions that learners ask themselves in order to achieve the essential learnings:

- How do I use *rectangular arrays* * and number patterns to understand, find and explain **prime and composite numbers*?
- How do I use **powers of ten* and square numbers as tools to understand the meaning and find the value of exponential expressions?
- How do I use my understanding of **prime numbers* and **exponents* to find the prime factorization of a number and express it using exponents?
- How do I recognize and express **equivalent fractions* and fractions in simplest form?
- How do I explain and use strategies to compare and order fractions with unlike denominators?
- What is similar about finding the LCM to write equivalent fractions and using the GCF to find the equivalent simplest form of a fraction?
- How do I use number lines, models, words and numbers to explain and show mixed numbers and their equivalent fractions?
- How is the inverse relationship of multiplication and division used to translate between equivalent mixed numbers and fractions?
- **How do I recognize and write equivalent representations of fractions and mixed numbers?*
- How do I use benchmarks of 0, $\frac{1}{2}$, and 1 to order and compare fractions and estimate sums and differences?
- **How do I use my understanding of basic operations with whole numbers to make meaning of addition and subtraction with fractions?*
- How do I translate my actions using models, pictures, and manipulatives to solve addition and subtraction problems with fractions into equations?
- How do I use my understanding of the meaning of the numerator and denominator of a fraction to regroup when adding and subtracting fractions?
- * *Presented in previous grade(s)*

Resources: Van de Walle, Chapters 15 (pp. 240-261); Ch. 16 (pp. 264-276); *Mathematics Source Book* (pp.59-70; 116)

**Harcourt Math – Grade 5
Number Theory; Fraction Concepts;
Addition and Subtraction with Fractions:
Chapters 15, 16, 19, 20,**

MODULE 7

Key Mathematical Concepts:

- Understand the meaning of and how to find: greatest common factors and least common multiples of a pair of numbers-
- Understand the meaning of prime and composite numbers-
- Know how to accurately evaluate expression with exponents-
- Understand the meaning and use of exponents in prime factorization-
- Understand that numbers written in exponential forms show a base raised to an exponent-
- Understand and know how to write equivalent fractions and fractions in simplest form and recognize characteristics of these fractions-
- Understand how to find the point on the number line for decimals, fractions, mixed numbers, and positive and negative integers-
- Use different strategies to convert a fraction to a decimal-
- Use and understand different strategies for finding equivalent fractions-
- Develop strategies for comparing fractions-
- Develop strategies for rounding numbers and use them in determining the “reasonableness of an answer”-
- Understand that decimal numbers (numbers less than 1) and fractions name parts of a whole-
- Fractions of the same value can be represented in many ways by writing equivalent fractions.
- A whole number can be represented as a fraction with a denominator of 1.
- Using benchmarks such as 0, $\frac{1}{2}$, 1, can help in estimating fraction sums and differences.
- The denominator remains the same when adding and subtracting fractions that have the same denominators.
- The least common multiple of the denominators of unlike fractions (LCD) can be used to rename them as like fractions (becomes the new denominator).
- Regrouping (renaming) a mixed number in subtraction is done when the numerator of the fraction part of the greater mixed number is less than the numerator of the lesser mixed number ($2\frac{6}{8} \neq 1\frac{7}{8}$).

Harcourt Mathematics

Grade 5

Module 7: Operations with Fractions
Chapters 19, 20, 21, and 22**MODULE 7 NOTES**

- Operations with fractions build on many previous fraction skills and concepts (GCF, LCM, equivalent fractions, mixed and improper fractions, “simplifying,” etc.). Many students will need additional experiences with these skills and concepts to be successful with fraction operations. The Routines section after each lesson offers suggestions for revisiting and practicing these skills and concepts.
- The Fraction Kit referred to in the module is the same one presented in the Harcourt training. It is strongly suggested for use in this module (students will need to add thirds, sixths, ninths, and twelfths as they work through the module). Some of the problems in the lesson may differ from problems in the book to match the denominators in the Fraction Kit.
- The vocabulary used in this module can be difficult all learners, especially English Language Learners. Reinforce fraction vocabulary with a chart as suggested on p. 344B, “English Language Learners.”
- Lesson plans were not provided for problem-solving lessons 19.7, 20.5; these lessons do not focus on the Key Standards for the grade level. (Mathematical Reasoning 2.1 – (estimation) is emphasized in this unit.) Writing their own problems deepens student understanding of concepts and enhances their ability to choose the correct operation for a given problem.

Operations with Fractions
25 Days of Instruction: Chapters 15, 16, 19, and 20

Day 1 CHAPTER 15: Number Theory Lesson 15.5 Prime and Composite Numbers	Day 2 Lesson 15.6 Introduction to Exponents	Day 3 Lesson 15.7 Evaluate Expressions with Exponents	Day 4 Lesson 15.2 Multiples and Least Common Multiples	Day 5 Lesson 15.1 Divisibility
Day 6 Lesson 15.3 Greatest Common Factor	Day 7 Lesson 15.8 Exponents and Prime Factors	Day 8 CHAPTER 16: Fraction Concepts Lesson 16.1 Relate Decimals to Fractions	Day 9 Lesson 16.2 Equivalent Fractions	Day 10 Lesson 16.3 Compare and Order Fractions
Day 11 Lesson 16.4 Simplest Form	Day 12 Lesson 16.5 Understanding Mixed Numbers	Day 13 Lesson 16.6 Problem Solving Strategy: Make a Model	Day 14 Performance Assessment: 5.5 Clowning Around *Strongly Recommended	Day 15 CHAPTER 19: Add and Subtract Fractions Lesson 19.1 Add and Subtract Fractions
Day 16 Lesson 19.2 Hands On: Add Unlike Fractions	Day 17 Lesson 19.3 Hands On: Subtract Unlike Fractions	Day 18 Lesson 19.4 Estimate Sums and Differences	Day 19 Lesson 19.5 Use Least Common Denominators	Day 20 Lesson 19.6 Add and Subtract Unlike Fractions
Day 21 CHAPTER 20 Add and Subtract Mixed Numbers Lesson 20.1 Add Mixed Numbers	Day 22 Lesson 20.2 Subtract Mixed Numbers	Day 23 Lesson 20.3 Hands On: Subtraction With Renaming	Day 24 Lesson 20.4 Practice with Mixed Numbers	Day 25 Assessment Chapters 19 & 20

DAY: 1
 CHAPTERS 15 and 1-2
 Number Theory, Using Whole Numbers and Decimals
 LESSON 15.5

MATERIALS:	16 Color tiles per pair; Grid paper;
LESSON FOCUS:	Identify: Prime and Composite Numbers
CALIFORNIA STANDARDS:	<p>Number Sense: 1.4: Determine the prime factors of all numbers through 50 and write the numbers as the product of their prime factors by using exponents to show multiples of a factor. Mathematical Reasoning: 2.3, 3.3</p>
PURPOSE OF LESSON:	Understand that whole numbers other than 1 are described as either prime or composite depending on their factors. The number 1 is neither prime or composite, it called a unit and is not included as part of prime factorization. Composite numbers have are the number itself, one and any other number as factors. A number is prime if it has exactly two factors – 1 and itself.
<p>LAUNCH:</p> <p>Bags of Tiles – 16 tiles per pair: Allow a few minutes for exploration with the tiles if this is the students' initial experience with this material-</p>	<p>Emphasize the concept of numbers having more than the factors of 1 and itself: Students build a 3X4 array. Use models to show that 3X4 can be turned to show 4X3, which is counted as a different array even though it is congruent and has the same factors.</p> <ul style="list-style-type: none"> • Discuss the array as the representation of: Factor X Factor = Columns X Rows = Product <p>Arrays using 6 tiles: Students build all the possible arrays.</p> <ul style="list-style-type: none"> • Discuss: How did you know when you found all of the arrays for 6? • Explain how the arrays showed the factors of 6. • Repeat for 5: Can you think of another number that will have only two factors? • Introduce and define the terms: prime and composite. Students show all the arrays for number 1 and discuss whether it fits the definition of prime or composite. (1 is the only number with 1 factor and it is neither prime nor composite). <p>Pg. 265: Connect: Make this chart with students (students need to create a chart for recording of Explore task) and complete it for the numbers 5 and 6.</p>

<p>EXPLORE: Tiles and Grid Paper</p>	<p>Arrays for 2-12: Groups build and arrange all the possible arrays for the numbers 2-12 and record on their charts and outline on grid paper.</p> <ul style="list-style-type: none"> • If groups finish early: Find the arrays for numbers 13-20 and extend charts. <p>Discussion: How did you know when you had built all the arrays for a number? Students use charts to verify definitions of prime and composite. What observations/did you notice about prime and composite numbers? (All of the arrays for prime numbers have only 1 and the number itself as factors. All prime numbers are odd except 2. All even numbers have 2 has a factor. All composite numbers have more than 1 and itself as factors.)</p> <p>Pg. 265: Reasoning Question: Explain why 2 is the only even prime number.</p>
<p>PRACTICE:</p>	<p>Pg. 265: #s: 6-10; 16 and 18. Draw arrays to prove that numbers are prime or composite. Discuss solutions with students.</p>
<p>SUMMARIZE:</p>	<p>Pg. 265: Assess: Discuss with a partner: Is 17 a prime number or a composite number? Explain how you decide if a number is prime or composite. Share and chart responses with whole class.</p>
<p>HOMEWORK:</p>	<p>Pg. 265: Assess: Write: Compare 10 and 11. List the factors for each number and determine which is prime and which is composite.</p> <ul style="list-style-type: none"> • Compare 23 and 24: Draw arrays and list factors for each. Explain which is prime and which is composite.

DAY: 2
 CHAPTERS 15 and 1-2
 Number Theory, Using Whole Numbers and Decimals
 LESSON 15.6

MATERIALS:	*Hundred Chart; Tiles/Grid Paper; * Transparency of Page 266-Lesson 15.6;
LESSON FOCUS:	Introduction to Exponents
CALIFORNIA STANDARDS:	Number Sense: 1.3: Understand and compute positive integer powers of nonnegative integers; compute examples as repeated multiplication. Algebra and Functions: 1.2 Mathematical Reasoning: 1.1, 2.3, 2.4, 3.3
PURPOSE OF LESSON:	Understand how to express numbers as powers of 10 as a bridge to the meaning and use of multiples and exponents. Recognize that a number written in exponential form shows a base raised to a number called an exponent.
LAUNCH: 1-50 on Hundred Chart can be used instead of writing the numbers 1-50	HW: Discuss solutions. Pg. 264B : Alternative Teaching Strategies – Sieve of Eratosthenes from Lesson 15.5. <ul style="list-style-type: none"> • Discuss: How is the Sieve of Eratosthenes similar to using arrays to find prime and composite numbers? • Discuss why this method finds all the prime numbers.
EXPLORE: *Transparency of Lesson 15.6 – Page 266 Tiles – 1 bag of 40 per group	Pg. 266: Quick Review: Discuss solutions- Students have books open or use *Transparency of Pg. 266. <ul style="list-style-type: none"> • Introduce square numbers (connect to Lesson 15.5) and exponential notation with Pg. 271: Use Thinker’s Corner- #1 to introduce vocabulary: <i>exponent, base, square number</i>. • Discuss: equivalent representations: <i>equal factors, exponent form</i> and <i>standard form</i> used in the chart on Pg. 271. <p>Students use tiles with small group to show squares (1X1 through 6X6) and complete the chart.</p> <ul style="list-style-type: none"> • Use solutions from Quick Review to introduce $10^3, 10^4, 10^5$ See Pg. 266: Examples in Learn Section.

PRACTICE:	Pg. 267: #s: 1-3; 7-8 and 34: Do with students. Pg. 267: #s: 26 – 36: Students work with partners and teacher confers with small groups/partners. Discuss solutions and justify solutions.
SUMMARIZE:	Pg. 267: Assess: Discuss and chart responses. Pg. 267: Write – record on chart.
HOMEWORK:	Pg. #s: 38-40

DAY: 3
 CHAPTERS 15 and 1-2
 Number Theory, Using Whole Numbers and Decimals
 LESSON 15.7

MATERIALS:	1 Sheet of paper for each student;
LESSON FOCUS:	Evaluate Expressions with Exponents
CALIFORNIA STANDARDS:	Number Sense: 1.3: Understand and compute positive integer powers of nonnegative integers; compute examples as repeated multiplication. Mathematical Reasoning: 1.1, 2.2, 2.4, 3.0
PURPOSE OF LESSON:	Understand exponential expressions and know that the base in an exponential expression is the number used as the factor. The exponent tells the number of factors of the base. Know how to find the value of the expression by multiplying the base by itself the number of times indicated by the exponent.
LAUNCH:	Pg. 268: Quick Review: Discuss solutions Pg. 268: Read: Learn – “One, Two, Four, Eight – Splitting of the Amoeba” problem. Students draw the first three stages of the amoeba. First - give background about amoeba. Use: Pg. 268: Teach – Guided Instruction Questions to discuss problem.
EXPLORE: 1 Sheet of paper for each student	Pg. 269: Powers of 2 - Paper Folding Activity: Students fold paper and count layers. Complete chart together or with partners and share results with class. Discuss with partner and then with table: <ul style="list-style-type: none"> • How is the splitting of the amoebas like the paper-folding task? • How did the number of layers change with each fold? • How many folds are needed to make 8 layers? • Can you figure the number of folds for any number of layers? Pg. 271: Thinker’s Corner: #2: Complete the chart with a partner. <ul style="list-style-type: none"> • How can you show 23 with your model? • Can you model 3 to the fourth power with a cube? Why or why not?
PRACTICE:	Pg. 270: #s 69 - 72: Verify the solution with your group. Discuss with whole class. Pg. 270: #s: 61 – 68. Discuss strategies for finding solutions. Students make up exponential expressions and show equivalent forms: In words, using equal factors form and in standard form. What is the value of “n” in the expression: $2^n = 8$? Explain your reasoning/strategy for finding the value of “n”.

SUMMARIZE:	<p>Pg. 271: Assess: Discuss (Refer to Thinker’s Corner cube pattern) “How many 1 inch cubes would you need to model/make a cube with 10 inches on a side? Discuss with a partner.</p> <ul style="list-style-type: none">• Explain how you found your solution.• Write the equivalent exponential, equal factors and standard forms to represent the total number of cubes in the model. <p>Pg. 271: Write: Explain how you would rewrite 5^4 as a multiplication problem and find its value.</p>
HOMEWORK:	Pg. 270: #s: 27–30 and 33-37,

DAY: 4
 CHAPTERS 15 and 1-2
 Number Theory, Using Whole Numbers and Decimals
 LESSON 15.2

MATERIALS:	Hundred Chart - * 1 copy per student; Number Line; Transparency 15.2
LESSON FOCUS:	Hands on: Multiples and Least Common Multiples
CALIFORNIA STANDARDS:	Mathematical Reasoning: 2.0 2.4: Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.
PURPOSE OF LESSON:	Understand that the least common multiple is the smallest quantity that is exactly divisible by each of two or more numbers. Example: 12 is the least common multiple of the numbers 2, 3, 4 and 6.
LAUNCH:	<p>Pg. 256: Skip count by 10's and 30's to 300 and record counting patterns.</p> <ul style="list-style-type: none"> • Introduce term: multiple: Connect term to skip counting patterns. Discuss: What are the multiples of 10? What are the multiples of 30? • Are there multiples of 10 that are also multiples of 30? • Explain/justify your response. <p>Pg. 256B: Writing in Mathematics Problem: If you want to serve 40 hotdogs, how many packages of buns are needed?</p> <ul style="list-style-type: none"> • How many packages of hotdogs? • Explain how you found your solution. <p>Pose a similar problem for eggs and muffins.</p>
EXPLORE: Hundred Chart - *1 copy per student Number Line	<p>Multiples on the Hundred Chart: Circle the multiples of 2 on the chart. Use a different color or symbol to circle the multiples of 3, 5, 6 and 10.</p> <ul style="list-style-type: none"> • Use this chart to find all the common multiples of: 2 and 3; 2, 5, 3 and 6, etc. • Discuss the meaning of “common”. <p>Pg. 257: Connect: Multiples on a Number Line: Use number line to identify common multiples. Discussion: What do you notice about the patterns?</p> <ul style="list-style-type: none"> • How do you know when you have found a common multiple of two numbers? • What do you do to find the common multiple of three numbers? • What multiples do 2 and 3 have in common? <p>Discuss: Can you always find a common multiple of two numbers by multiplying them together? Does this give the Least Common Multiple? Explain your thinking/reasoning.</p>
PRACTICE:	<p>Pg. 257: #s: 4, 8 - 11. Work with partner/individually. Show how you found each solution. Discuss solutions with class.</p>

SUMMARIZE:	Discuss with partner before whole class discussion: What is a multiple? What are different ways to find multiples? <ul style="list-style-type: none">• How do you find common multiples? Write the first 6 multiples of 3 and 8. Find the LCM and explain your strategy for finding it.
HOMEWORK: Transparency 15.2	Pg. 257: Assess: Lesson Quiz

