

SAN DIEGO CITY SCHOOLS

Instructional Module to Enhance the Teaching of

HARCOURT MATH

California Edition

GRADE 6

Module 5

Algebra: Expressions and Equations

KEY MATHEMATICAL CONCEPTS FOR GRADE SIX - BIG IDEAS:

By the end of grade six, students will:

- Master the four arithmetic operations with positive and negative whole numbers, positive fractions, and decimals. They accurately solve problems involving fractions, ratios, proportions, and percentages.
- Understand the concepts of mean, median and mode of data sets, and how to calculate the range. They analyze data and sampling processes for possible bias and misleading conclusions. They calculate the probabilities for compound events and understand the difference between independent and dependent events.
- **Write verbal expressions and sentences as algebraic expressions and equations; they evaluate algebraic expressions, solve simple linear equations, and graph and interpret their results.** They analyze and use tables, graphs, and rules to solve problems involving rates and proportions.
- Deepen their understanding of the measurement of plane and solid shapes and use this understanding to solve problems.

Key Mathematical Concepts for Module 5

- Translate word expressions into algebraic expressions and models, using variables to represent unknowns.
- Translate between words, algebraic expressions, models and graphical representations.
- To evaluate algebraic expressions:
 - Combine like terms.
 - Replace variables with numbers
 - Use order of operations and associative and distributive properties.
- To solve algebraic equations:
 - Use addition, subtraction, multiplication and division properties of equality to isolate the variable and solve equations.
 - Use the same strategies for solving equations when solving problems with formulas.

Chapter 14: Expressions

Lesson 14.1: Write Expressions

Lesson 14.2: Evaluate Expressions

Lesson 14.3: Squares and Square Roots

Lesson 14.4: Expressions with Squares and Square Roots

Chapter 15: Addition and Subtraction Equations

Lesson 15.1: Connect Words and Equations

Lesson 15.2: Model and Solve One-Step Equations

Lesson 15.3: Solve Addition Equations

Lesson 15.4: Solve Subtraction Equations

Chapter 16: Multiplication and Division Equations

Lesson 16.1: Model Multiplication Equations

Lesson 16.2: Solve Multiplication and Division Equations

Lesson 16.3: Use Formulas

Lesson 16.4: MATH LAB: Two-Step Equations (*Optional*)

Lesson 16.5: Problem Solving Strategy: Working Backward

MODULE 5: UNIT 5: ALGEBRA: EXPRESSIONS AND EQUATIONS

Chapter 14: Expressions

DAY: 1 - LESSON: 14.1

LESSON FOCUS:	Write and Model Expressions
CA STANDARD:	AF 1.0; AF 1.2; (MR 1.1; MR 2.5; MR 3.2)
Purpose of Lesson: Math Background: TE p. 274A	At the end of the lesson students will be able to translate between words, concrete models* and algebraic expressions. <i>*Modification: Students are using algebra tiles before Lesson 15.2 to provide more opportunities for them to make connections between the representations.</i>
Warm-up/ Routine(s):	Mental Math: Blank Number Line: Start with -3; Go by 1/2: Give time for students to get to at least +2. The number line is used as a tool to solve these problems if students need it: $-2\frac{1}{2} + \frac{3}{4}$; $1 - 1\frac{1}{2}$; $1 + (-1\frac{1}{2})$ <i>(Note that subtracting a positive is equivalent to adding a negative.)</i>
LAUNCH: <u>Materials:</u> Tongue depressors may be used as the variable in place of cardstock algebra tiles (TR21). Two-sided counters or algebra tile squares may be used in place of index cards with numbers 0-9.	ALTERNATIVE TEACHING STRATEGY: TE p. 274B: Modified: <ul style="list-style-type: none"> • Review: Basic operations are not performed when writing expressions, only when evaluating them. • Introduce the tongue depressor or algebra tile rectangle as a variable that can represent an unknown number. <i>"If this stick/rectangle represents the cost of one ticket, how would I represent 4 times the cost of a ticket?"</i> ($4t$) • Build the models together for <i>"3 dollars above the cost"</i>, <i>"two dollars less than the sale price"</i>, and <i>the product of 3 and the number of people"</i>. Translate the models into algebraic expressions with the students. ($c + 3$; $y - 2$; $3n$) • Write algebraic expressions $2b + 4$ and $3z - 3$ and ask the students to translate the algebraic expressions into models. Translate into words.
EXPLORE: Refer to Independent Practice: TE p. 275: Algebraic Thinking	Groups/Partners: <ul style="list-style-type: none"> • Partners/groups build models and write algebraic expressions for these situations (discuss after each one or after two or three depending on the need of the class): <ul style="list-style-type: none"> • <i>"2 dollars less than the cost" and "double the cost plus four dollars".</i> • <i>"4 less than the product of four times the cost."</i> • <i>"The sum of two times the distance and three more."</i> • When the number is greater than nine, have students write the numeral instead of building with the unit counters or the unit squares. • p. 275: #7-8 and #13-14: Translate only into algebraic expression from words.
Practice: See English Language Learners: TE p. 274B.	<ul style="list-style-type: none"> • p. 274: Lawn mowing problem: <i>Have students translate the word problem into an algebraic expression and discuss.</i> • p. 275: Choose from #19, #20 and #21
SUMMARIZE/Closure TE p. 275; Lesson Quiz: 14.1	<ul style="list-style-type: none"> • Chart the algebraic expressions from the problems. Include sketches of the models for the first ones. These expressions will be used in the next lesson. • Questions to Discuss: <ul style="list-style-type: none"> • <i>"How do you know when to use a variable in an expression?"</i> • <i>"Describe a situation to your partner that can be represented by the expression $3y + 5$, and sketch/build an equivalent model."</i> • TE p. 275; Lesson Quiz: 14.1: Select Problems.
Homework:	<ul style="list-style-type: none"> • p. 275: #10-12 and #16-18 or • CHALLENGE 14.1 Transparency: MIXED REVIEW AND TEST PREP: p. 275

MODULE 5: UNIT 5: ALGEBRA: EXPRESSIONS AND EQUATIONS

Chapter 14: Expressions

DAY: 2 - LESSON: 14.2

LESSON FOCUS:	Write and evaluate algebraic expressions																
CA STANDARD:	AF 1.2; AF 1.3; (AF 1.0; NS 2.3; MR 2.2; MR 2.5; MR 3.2)																
Purpose of Lesson: Math Background: TE p. 276A	At the end of the lesson students will be able to write and evaluate algebraic expressions. They will translate between words, concrete models and algebraic expressions.																
Warm-up/ Routine(s): TE p. 276	Mental Math: Quick Review p. 276: <i>Evaluate</i> #1 and 2 for $x = 3$ and $y = 4$; then evaluate #3 and 4 for $x = -2$ and $y = -4$ <i>Record</i> numerical <i>paths</i> that match student explanations.																
LAUNCH: <u>Books Closed</u> <i>Look for opportunities to use "terms" and "like terms" when asking questions and discussing expressions and equations.</i>	<p>Combine Like Terms (p. 277):</p> <ul style="list-style-type: none"> <i>Display</i> the chart to discuss "terms" and "like terms": Ask students to determine what is happening in the table first using Private Think Time and then partner/group. <i>Discuss</i> what they notice about "terms" and make explicit the meanings of terms and like terms. (p. 277). <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th style="text-align: left;">Algebraic Expression</th> <th># of Terms</th> <th>Like Terms</th> <th>Simplified</th> </tr> </thead> <tbody> <tr> <td>$6x + 5x + 17$</td> <td>3</td> <td>$6x$ and $5x$</td> <td>$11x + 17$</td> </tr> <tr> <td>$2 + 3y + 4y - 5$</td> <td>4</td> <td>$3y$ and $4y$ 2 and -5</td> <td>$7y - 3$</td> </tr> <tr> <td>$42 + 3a - 2a + 3 - 1$</td> <td>5</td> <td></td> <td></td> </tr> </tbody> </table> <ul style="list-style-type: none"> Add a couple of expressions with exponents for students to use in the chart. Have each group evaluate one algebraic expression and its equivalent simplified expression using a number under 5; note the equivalent solutions. <p>"What is the advantage to combining like terms before evaluating expressions?"</p>	Algebraic Expression	# of Terms	Like Terms	Simplified	$6x + 5x + 17$	3	$6x$ and $5x$	$11x + 17$	$2 + 3y + 4y - 5$	4	$3y$ and $4y$ 2 and -5	$7y - 3$	$42 + 3a - 2a + 3 - 1$	5		
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$2 + 3y + 4y - 5$	4	$3y$ and $4y$ 2 and -5	$7y - 3$														
$42 + 3a - 2a + 3 - 1$	5																
EXPLORE: p. 278: #17-20	<p>p. 278: #17-20:</p> <ul style="list-style-type: none"> <i>Assign</i> a different starting problem for each group. Students <i>simplify</i> and then <i>evaluate</i> the expressions, using order of operations. Students <i>record</i> the expressions and the <i>numerical path</i> used to simplify and evaluate. Choose two groups to <i>record</i> their starting problem and discuss during the Summarize. 																
Practice: TE p. 272G	<p>TE p. 272G: Practice Game: Evaluation Elevator (Play with partners vs. partners or individual vs. individual. There are more opportunities for discourse with partners.)</p> <p><i>Modify</i> game by using different numbers on the spinner or number cube.</p>																
SUMMARIZE/Closure:	<i>Discuss</i> problems on board. ASSESS: <i>Discuss:</i> TE p. 279 Lesson Quiz: 14.2: TE p. 279 #3 and #4																
Homework:	<ul style="list-style-type: none"> MIXED REVIEW AND TEST PREP: p. 279 Play the Elevator Game with someone at home. 																

MODULE 5: UNIT 5: ALGEBRA: EXPRESSIONS AND EQUATIONS

Chapter 14: Expressions

DAY: 3 - LESSON: 14.3

LESSON FOCUS:	Squares and Square Roots
CA STANDARD:	NS 2.0 Key Standard; (MR 2.4)
Purpose of Lesson: Math Background: TE p. 282A	At the end of the lesson students will be able to use a model to find squares and square roots. They will generalize the formulas for finding area and perimeter of a square and its relationship to square root.
Warm-up/ Routine(s): Base 10 paper and or pieces should be available.	Mental Math: 28×4: <ul style="list-style-type: none"> Watch for the distributive property and compensation: $(25 \times 4) + (3 \times 4)$ and $(30 \times 4) - (2 \times 4)$ or $(28 + 2) \times 4 - (2 \times 4)$ Discuss the mental strategies for solving the first problem before showing the other two. Assign different starting problem to groups: 2.8×-4; -2.8×4: If the strategy of estimating (3×4) does not come up for placement of the decimal point, remind the students.
LAUNCH: <i>*Note: Students often confuse linear (dimension) with area. Emphasize that the square root is length and does not have any area.</i>	Students build a square using 9 tiles: <ul style="list-style-type: none"> Have students make observations about the square to a partner. Discuss as a class. If perimeter, dimension, area, equal sides don't come up, bring them up. * Have students run fingers down the edge of the three linear units of the square to illustrate that dimensions are linear, or line, not area. Introduce the term square root and its symbol. "A tree grows from a root; a square is built from its square root." Write with symbols: $\sqrt{9} = 3$ and 3^2 (squared) = 9 Find the perimeter and area of the square. Set up Graph: "Relationship between Square Root and Perimeter" <ul style="list-style-type: none"> Label the x axis, Square Root and the y axis, Perimeter: Review graphing an ordered pair (3, 12) for the perimeter of 3 squared.
EXPLORE:	Make a table labeled: Square Root; Perimeter; Area: <ul style="list-style-type: none"> Students draw squares from square roots that are consecutive whole numbers (Draw the dimension, or square root, on graph paper; then enclose the square.) and record the data on the table. Emphasize that the square root is the length of the dimension of the square. Graph the ordered pairs for the square roots/perimeter relationship. "What do you notice about the graph?"
Practice: Tools: Calculators	p. 281: <ul style="list-style-type: none"> Use a calculator to find square roots and to square numbers. Rewrite problems 1-7 following the pattern of #7. Use Frame: If $\underline{\quad} \times \underline{\quad} = \underline{\quad}$, then $\sqrt{\underline{\quad}} = \underline{\quad}$
SUMMARIZE/Closure: <i>Emphasize translating between representations.</i>	<ul style="list-style-type: none"> Dictate expressions involving numbers squared and square roots. Students write them symbolically. Then have partners read symbolic expressions to partners in words, reversing the translation. Students write rules in words and symbols for finding the area and perimeter of any square. Discuss and record in words and symbols. Students write an explanation of square roots to someone who has never studied them. They should include diagrams and symbolic expressions. (Finish for homework and use as an assessment.)
Homework:	MIXED REVIEW AND TEST PREP: p. 281 and p. 278, #8, #9, #10

MODULE 5: UNIT 5: ALGEBRA: EXPRESSIONS AND EQUATIONS
Chapter 14: Expressions

DAY: 4 - LESSON: 14.4

LESSON FOCUS:	Expressions with Squares and Square Roots
CA STANDARD:	NS 2.0 Key Standard; AF 1.3; (MR 2.5)
Purpose of Lesson: Math Background: TE p. 282A	At the end of the lesson students will be able to solve expressions involving squares and square roots.
Warm-up/ Routine(s): TE p. 282A <u>Materials:</u> Grid paper in plastic "white board"	PROBLEM OF THE DAY 14.4: TE p. 282A (Students may sketch on grid paper to access problem). Use vocabulary from Day 3 when discussing solution.
LAUNCH: TE p. 282A	NUMBER OF THE DAY 14.4: TE p. 282A: <ul style="list-style-type: none"> • Modification: Have students first write an algebraic expression to represent the problem; use z for the variable, age. Review the order of operations and include new order: <u>Squares roots are evaluated at the same time as exponents.</u>
EXPLORE: p. 283	<ul style="list-style-type: none"> • Partners evaluate expressions: p. 283, #2, #3, #8, #9 • Split up the problems and assign different starting problems to partners. (<i>Students will only complete one or two problems.</i>) • <i>Partners explain solution to another pair in the group who did not solve the same problem.</i> • Complete either #14 or #15 and #16
Practice: Order of Operations Spinner Game (attached)	"Order of Operations" Spinner Game (<i>See description at the end of lesson.</i>) <ul style="list-style-type: none"> • Model one round before playing. • Students record some of the equations on the board.
SUMMARIZE/Closure: TE p. 283	<ul style="list-style-type: none"> • Partners read aloud the expressions in Lesson Quiz: 14.4, TE p. 283; translating the symbolic to words. • Discuss solutions to p. 283 #16 and one or two other problems. • Lesson Quiz: 14.4: <i>Choose two or three problems.</i>
Homework:	<ul style="list-style-type: none"> • MIXED REVIEW AND TEST PREP: p. 283 • p. 283: #10-13

"Order of Operations" Spinner Game*

*This game is adapted from materials developed by Clyde Yoshida

Use "Order of Operations" Spinner Tool that is attached at the end of the module:
(Numbers given are just suggestions. Spinners with different fractional parts can also be used.)

- *In the sections of the first circle spinner, students write (on plastic sleeve):
 $\sqrt{25}$, $\sqrt{36}$, $\sqrt{64}$, $\sqrt{9}$ (These can be changed.)*
- *In the second circle, write the numbers 1-8. (Numbers can be changed and some can be negative.)*
- *In the third circle write both positive and negative integers using numbers lower than 10.*

Directions:

- *Person/team spinning first chooses one of the two frames below the spinners.*
- *Spin from the first spinner and write the number in one place on the equation frame. Continue with the other two spins, place the number and evaluate the expression.*
- *The second person/team spins and places their numbers in the second frame and evaluates.*
- *Record each equation on a paper. Keep track of the cumulative scores on a number line. (The game could be continued over time because the number line would always contain the running total.)*

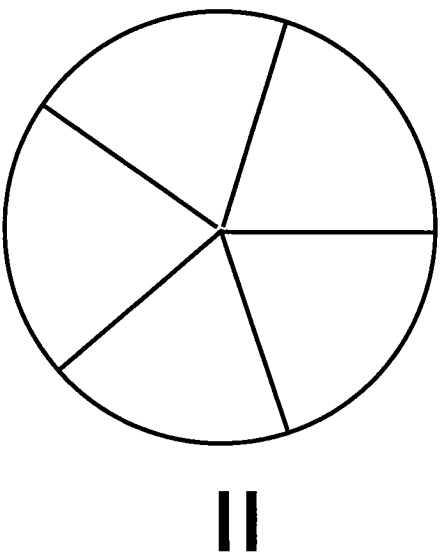
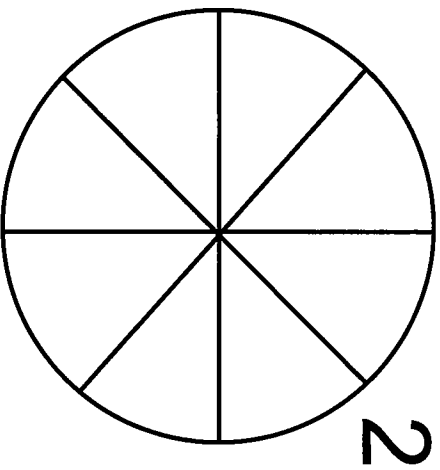
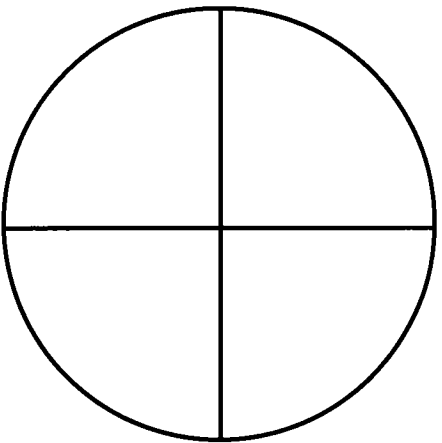
Object of Game:

- *Different strategies surface if the object is to have the lowest absolute value (the number closest to zero), at the end.*
- *Instead of the lowest absolute value, the winner could be person with the highest score (Net Value), or lowest score (-6 would beat 3).*

Operation signs are determined by the number frame chosen.

See next page for Spinner Game Board.

ORDER OF OPERATIONS: SPINNER GAME



$$\underline{\hspace{2cm}} + \underline{\hspace{2cm}} \times \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$\left(\underline{\hspace{2cm}} + \underline{\hspace{2cm}} \right) \times \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

MODULE 5: UNIT 5: ALGEBRA: EXPRESSIONS AND EQUATIONS
Chapter 15: Addition and Subtraction Equations

DAY: 5 - LESSON: 15.1

LESSON FOCUS:	Connect Words and Equations
CA STANDARD:	AF 1.0; AF 1.1 Key Standard; (MR 2.5)
Purpose of Lesson: Math Background: TE p. 288A	At the end of the lesson the student will be able to translate words into numbers, variables and operations.
Warm-up/ Routine(s):	Mental Math: $2 \frac{1}{4} + (-\frac{1}{4})$; $-2 \frac{1}{2} + \frac{1}{2}$; $-2 \frac{1}{4} + (-\frac{1}{2})$ Students may use tools to scaffold. (number line, sketches, fraction bars, rulers)
LAUNCH: Reference: TE p. 288 Guided Instruction <u>Books Closed:</u> p. 289	Create a chart for Expressions and Equations <i>List</i> three or four under each and ask students to discuss difference between equations and expressions. <i>Ask:</i> "How can we turn an equation into an expression?" "An expression into an equation?" Students write equations that are equivalent to the word sentences the teacher reads aloud: p. 289; #3-12 <i>Discuss</i> words/phrases that are equivalent to the equal sign, the addition sign; the subtraction sign; the multiplication sign and division sign. <i>Chart</i> them as they come up in the lesson.
EXPLORE: <u>Materials:</u> Challenge 15.1: 1 per 2 students;	CHALLENGE 15.1: TE p. 289 <ul style="list-style-type: none"> • <i>Assign</i> a starting problem for each group; then students may choose. • <i>Emphasize making sense</i> of a couple of the exercises by reading aloud in groups and translating the different equations into words before choosing. • <i>Verify</i> that the two choices result in equivalent solutions.
Practice:	Students write equations and translate them into word sentences.
SUMMARIZE/Closure: TE p. 289: Lesson Quiz: 15.1	Add to the chart on equivalent expressions for operation symbols started in Launch. <ul style="list-style-type: none"> • ASSESS: DISCUSS: WRITE: TE p. 289 • <i>Translate Lesson Quiz #6;</i> TE p. 289, into an equation with partner. <i>Discuss strategies.</i> • Lesson Quiz: 15.1: TE p. 289 #1-5
Homework:	MIXED REVIEW AND TEST PREP: p. 289

MODULE 5: UNIT 5: ALGEBRA: EXPRESSIONS AND EQUATIONS

Chapter 15: Addition and Subtraction Equations

DAY: 6 - LESSONS: 15.2; 15.3; 15.4

LESSON FOCUS:	Model and Solve One-Step Equations
CA STANDARD:	AF 1.0; AF 1.1 Key Standard; NS 2.0 Key Standard; (MR 2.5)
Purpose of Lesson: Math Background: TE p. 291A and 294A	At the end of the lesson students will be able to model and solve one-step addition and subtraction equations.
Warm-up/ Routine(s):	NUMBER OF THE DAY 15.3: TE p. 291A
LAUNCH: <i>Materials:</i> Algebra tiles for all students (You may substitute tongue depressors for variable tile and two-sided counters for the units). Transparency and one copy per student of the "Balance Scale" attached to this module lesson.	Introduce "Balance the Equation": (See explanation and directions at end of the lesson.) <ul style="list-style-type: none"> • Model the activity with the class for a couple of rounds until students have modeled and solved an addition and subtraction equation. Have partners discuss moves before doing it whole class. • Ask: "How can I end up with just the variable on this side and still keep the balance?" (Take away the 2 units from both sides.) • Substitute the value of the variable into the equation and check.
EXPLORE: See page following (Day 6) for directions.	"Balance the Equation": <ul style="list-style-type: none"> • Partners solve the equations using the balance. • Record the path to the solution for each problem. Problems: $14 = x - 2$; $m + 3 = 14$; $n - 5 = 4$; $3 = n - 1$. Verify answers by substituting the solution for the variable.
Practice:	p. 290; #1-8. <u>Change the sign from + to - on all the even number problems so students are using both addition and subtraction properties of equality.</u> <ul style="list-style-type: none"> • Have some students record sketches of actions with the model on the board and equivalent actions with equations from Explore.
SUMMARIZE/Closure:	Students explain their models and equivalent representation with equations recorded on the board. <ul style="list-style-type: none"> • $t - 4 = 9$: <ul style="list-style-type: none"> • One student models each step using the algebra pieces at the overhead and talks about the reasoning behind the actions. • The class records algebraic representation that matches the actions. • Verify the answer by substituting the solution for the variable. • Students solve 2 equations and record a sketch of the action with the pieces and the equivalent steps represented algebraically: $x - 4 = 12$ and $14 = y + 6$
Homework:	<ul style="list-style-type: none"> • Students write 2 equations, 1 involving addition and 1 involving subtraction. Represent the solutions algebraically and with sketches. • p. 293, #9-10; and p. 295, #8-9

"Balance the Equation"

<p><i>Acknowledge that the students can use "Guess and Check" to figure out the answer, but they are to solve it algebraically. They will be able to use the same strategies when problems are more complex.</i></p> <p style="text-align: center;">Always translate the actions with the model into symbolic and numerical representations.</p> <p>For expressions involving subtraction, help students connect to what they know about making zero by adding the opposite.</p> <p>$x - 2 = 8$ is a variable piece and 2 red units that are equivalent to 8 yellow.</p>	<p>"Balance the Equation":</p> <ul style="list-style-type: none"> • <i>Students build a model on the left side of their balance scale (in plastic "white board") to represent this situation: "I had some money and then I got 2 more dollars."</i> • <i>Build and record the second side of the equation: "My money was equal to \$8." (Students should have a variable and two units on one side and 8 units on the other.)</i> • <i>"What do we know about a balance scale?" Establish the balance of the two sides as equivalent to the equal sign. Both sides have the same value.</i> • <i>Establish that we want to find out what the variable is, so it will be the variable on one side of the balance and its value on the other. Idea is to isolate the variable on one side but keep both sides equivalent. (Anything done to variable side has to be done to other side.)</i> • <i>Partner on the side without the variable starts:</i> <ul style="list-style-type: none"> • <i>Take away the 2 units.</i> • <i>Take away two units from other side.</i> • <i>Variable = 6.</i> • <i>Go back and record the steps. $x + 2 = 8$;</i> $\begin{array}{r} -2 \quad -2 \\ \hline x \quad = 6 \end{array}$ <p style="text-align: center;"><i>OR: $x + 2 = 8$; $x + 2 - 2 = 8 - 2$; $x = 6$</i></p> <ul style="list-style-type: none"> • <i>"I had some money and I spent \$2. Then I had \$8."</i> <ul style="list-style-type: none"> • <i>Ask students how they represent "spent \$2" with integers. (2 red)</i> • <i>Build the equation on the scale and write it. $x - 2 = 8$: variable piece and two red counters on one side and 8 yellow on the other.</i> • <i>Give students time to determine what to do on both sides of the balance and end up with x on one side. (Zero is the same as getting rid of the -2; add positive 2 to each side and result is $x = 10$.)</i> • <i>Some students will want to take away the 2 red and then will take away 2 yellow. Have them verify that they did exactly the same thing to each side. (They took away the same absolute value, 2 counters, but the net value was not the same.)</i>
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See next page for Balance Pan.

