



**SAN DIEGO CITY SCHOOLS**

---

**Instructional Module to Enhance the Teaching of**

# **HARCOURT MATH**

**California Edition**

**GRADE 6**

## **Module 7**

**Geometry, Spatial Reasoning and  
Measurement (Circles)**

## **KEY MATHEMATICAL CONCEPTS FOR GRADE SIX - BIG IDEAS:**

By the end of grade six, students will:

- Master the four arithmetic operations with positive and negative whole numbers, positive fractions, and decimals. They accurately solve problems involving fractions, ratios, proportions, and percentages.
- Understand the concepts of mean, median and mode of data sets, and how to calculate the range. They analyze data and sampling processes for possible bias and misleading conclusions. They calculate the probabilities for compound events and understand the difference between independent and dependent events.
- Write verbal expressions and sentences as algebraic expressions and equations; they evaluate algebraic expressions, solve simple linear equations, and graph and interpret their results. They analyze and use tables, graphs, and rules to solve problems involving rates and proportions.
- **Deepen their understanding of the measurement of plane and solid shapes and use this understanding to solve problems.**

### **Key Mathematical Concepts for Module 7**

- Classify triangles by their angles and sides.
- Use understanding of the relationships of complementary, supplementary and vertical angles to determine the measures of unknown angles.
- Generalize 180 degrees as the sum of the interior angles of any triangle and use this generalization to find:
  - The measure of an unknown angle in a triangle;
  - The sum of the interior angles in polygons with different numbers of sides.
- Translate between geometric words, symbols and drawings of geometric figures.
- Relate the perimeter of a polygon to the circumference of a circle.
- Generalize the estimate of the ratio of the diameter of any circle to its circumference; use the ratio to estimate the circumference of any circle.
- Use  $\pi$  to represent the exact ratio between the diameter and circumference and substitute 3.14 and  $\frac{22}{7}$  into the formulas for circumference and the area of a circle.

#### **Chapter 17: Geometric Figures**

Lesson 1: Points, Lines and Planes

Lesson 2: MATH LAB: Angles

Lesson 3: Angle Relationships

Lesson 4: Classify Lines

#### **Chapter 18: Plane Figures**

Lesson 1: Triangles

Lesson 4: Draw two-dimensional Figures

Lesson 5: Circles (*Optional at this time*)

#### **Chapter 25: Length and Perimeter**

Lesson 4: MATH LAB: Circumference

Lesson 5: Circumference

#### **Chapter 26: Area**

Lesson 3: MATH LAB: Area of a Circle

Lesson 4: Algebra: Areas of a Circle

**\*Chapter 19 can be found in Module 10.**

## MODULE 7: UNIT 6: GEOMETRY, SPATIAL REASONING AND MEASUREMENT (CIRCLES)

### MODULE 7: DAY 1 - LESSON: 17.1/ 17.2/ 17.3

<b>MATERIALS:</b>	Protractor, <b>IS284</b> (Intervention Strategies & Activities Book), one per student
<b>LESSON FOCUS:</b>	Angles and Angle Relationships
<b>CA STANDARD:</b>	<b>MG 2.2 Key Standard</b> ; MG 1.0; MG 2.0; MG 2.1; (MR 2.5)
<b>Purpose of Lesson:</b> Math Background: TE pp. 322A/326A	At the end of the lesson students will be able to name, measure and draw angles and describe geometric figures by using geometric terminology. Additionally, they will be able to use the understanding of the relationships of complementary and supplementary angles to find unknown angles.
<b>Warm-up/ Routine(s):</b> TE pp. 322-323 Reference p. 322	<ul style="list-style-type: none"> <li>• <b>PRACTICE 17.1; #1-6 (Make a transparency.) and/or</b></li> <li>• <b>Lesson Quiz 17.1; TE p. 323: Discuss the symbolic representations of naming segments and lines (instead of as a Quiz).</b></li> </ul>
<b>LAUNCH:</b>  IS284  Reference p. 327	<p><b>MODEL MEASURING ANGLES WITH A PROTRACTOR AT THE OVERHEAD:</b></p> <p>Use the terms <u>ray</u> and <u>vertex</u> when modeling.</p> <ul style="list-style-type: none"> <li>• <b>Discuss</b> the use of estimation in determining whether to use the top or bottom measure of the scale on the protractor.</li> <li>• <b>Compare</b> the classification of the angle to the exact measure to see if it makes sense. <b>IS284: #1-3.</b></li> </ul> <p><b>COMPLEMENTARY, SUPPLEMENTARY AND ADJACENT ANGLES:</b></p> <ul style="list-style-type: none"> <li>• <b>Sketch</b> several pairs of adjacent and non-adjacent complementary and supplementary angles under the headings of <u>Complementary Angles</u> and <u>Supplementary Angles</u>. (See top of page 327 for examples of non-adjacent complementary and supplementary angles without a common vertex and ray.)</li> <li>• Students use <i>Private Think Time</i>, and then partner talk to <b>make observations</b> about complementary and supplementary angles.</li> <li>• <b>Discuss their observations; clarify the meanings.</b></li> <li>• <b>Write <u>Adjacent Angles</u></b> and circle the adjacent supplementary and complementary angles. <b>Add sketches</b> of some examples of adjacent angles that are not complementary or supplementary.</li> <li>• <b>Repeat</b> observation/discussion process regarding adjacent angles.</li> </ul>
<b>EXPLORE:</b>  IS284	<p><b>PARTNER WORK: IS284: #4-9</b></p> <ul style="list-style-type: none"> <li>• Students <b>draw</b> one ray to each of the angles to create complementary or supplementary angles. <b>Label</b> the new ray with a point and a letter and include the arrow to show it is a ray. (Examples: For #5, drawing a new ray between rays BA and BC forms complementary angles or extending BC with a new ray forms supplementary angles.)</li> <li>• Use a protractor to <b>measure</b> only one of the complementary or supplementary angles for each example.</li> <li>• Determine the measure of the remaining supplementary or complementary angles without measuring.</li> </ul>
<b>Practice:</b>	<b>Record Algebraic Representations:</b>

	<ul style="list-style-type: none"> <li>• <b>Select</b> students to record their sketches of #4 and explain strategy used to find the measure of the unknown angle.</li> <li>• <b>Translate</b> the solution into an equivalent algebraic solution. (Example: <math>120 + y = 180</math>; <math>120 - 120 + y = 180 - 120</math>; <math>y = 60</math>)</li> <li>• Partners <b>record</b> algebraic representations for their solutions on <b>IS284</b>.</li> <li>• Pairs exchange papers and check the algebraic solutions that have been completed.</li> <li>• <b>Select</b> a few students to record algebraic solutions on the board for discussion during Summary.</li> </ul>
<p><b>SUMMARIZE/ Closure:</b></p>	<ul style="list-style-type: none"> <li>• <b>Discuss algebraic solutions.</b></li> <li>• Partners/ group, then whole class: <i>"How are the algebraic equations for finding unknown supplementary and complementary angles similar and different."</i></li> <li>• <b>Students classify angles:</b> <ul style="list-style-type: none"> <li>• <b>Write</b> acute, obtuse, right, straight, complementary, supplementary and adjacent on board.</li> <li>• <b>Sketch</b> a pair of adjacent complementary angles. Have students describe the angles using as many of the vocabulary words as they can that apply to the angles. For example: If one of the complementary angles were <math>50^\circ</math>, the angles would be complementary and adjacent. Both of the angles would also be described as acute. If one supplementary angle were <math>45^\circ</math>, one would be acute and one would be obtuse.</li> <li>• Partners alternate describing angles after one or two have been modeled with the class.</li> </ul> </li> </ul>
<p><b>Homework:</b></p>	<ul style="list-style-type: none"> <li>• <b>IS284 #10, 11, 12</b> (same as Explore and Practice with #4-9)</li> <li>• <b>p. 328, #4-9</b></li> </ul>

# MODULE 7: UNIT 6: GEOMETRY AND SPATIAL REASONING

## MODULE 7: DAY 2 - LESSON: 17.3

<b>MATERIALS:</b> TE p. 326B	2 strips of cardboard and a fastener for each pair (See <b>ALTERNATIVE TEACHING STRATEGY</b> ); protractor; straight edge; plastic "white boards"
<b>LESSON FOCUS:</b>	Angle Relationships
<b>CA STANDARD:</b>	<b>MG 2.2 Key Standard; MG 1.0; MG 2.0; MG 2.1; (MR 2.5)</b>
<b>Purpose of Lesson:</b> Math Background; TE p. 326A	At the end of the lesson students will be able to use the understanding of the relationships of vertical, complementary and supplementary angles to find unknown angles.
<b>Warm-up/ Routine(s):</b>  p. 326	<b>QUICK REVIEW: p. 326:</b> <i>Choose 2 or 3 and discuss strategies.</i>
<b>LAUNCH:</b>  TE p. 326B	<p><b>Vertical Angles:</b></p> <ul style="list-style-type: none"> <li>• <b>Sketch</b> the vertical angles (top of page 326) on the overhead or board.</li> <li>• <b>Point out</b> that vertical angles are formed opposite each other when two lines intersect, <u>but don't mention that vertical angles are equal or congruent.</u></li> <li>• "What do you notice about these vertical angles?" (If they notice that they look equal, ask how they could verify that they are congruent.)</li> <li>• Have each pair fasten the cardboard together with the fastener. (See <b>ALTERNATIVE TEACHING STRATEGY, TE p. 326B</b>)</li> <li>• Have pairs test different vertical angles for congruency. They can form identical vertical angles with another pair and flip one over to test for congruency.</li> </ul>
<b>EXPLORE:</b>  TE p. 326B	<ul style="list-style-type: none"> <li>• <b>ALTERNATIVE TEACHING STRATEGY: TE p. 326B</b></li> <li>• Students use their understanding of finding unknown supplementary angles and congruency of vertical angles to <b>find</b> all four angles.</li> <li>• Pairs <b>record</b> algebraic equations to match the solutions.</li> </ul>
<b>Practice:</b>	TE p. 326 <b>RETEACH 17.3</b>
<b>SUMMARIZE/Closure</b>	<b>Discuss Thinker's CORNER; p. 329</b>
<b>Homework:</b>	<b>p. 329; #24-27: <u>Modify</u> directions to include "estimate the measure of the numbered angles to the nearest 10°."</b>

# MODULE 7: UNIT 6: GEOMETRY AND SPATIAL REASONING

## MODULE 7: DAY 3 - LESSON: 17.4

<b>MATERIALS:</b> TR59 and TR60	TR59 and TR60: One copy of 1/2 page isometric dot and 1/2 page square dot paper for each student to use in plastic "white board."
<b>LESSON FOCUS:</b>	Classify Lines
<b>CA STANDARD:</b>	MG 1.0; (MG 2.2; MR 2.5)
<b>Purpose of Lesson:</b> Math Background; TE p.330A	At the end of the lesson students will be able to classify the different types of lines.
<b>Warm-up/ Routine(s):</b> TE p. 329	TE p. 329: Lesson Quiz 17.3
<b>LAUNCH:</b> p. 330	<p>Students draw lines on square or isometric dot paper to represent the verbal instructions. Introduce symbols for perpendicular and parallel and write symbolically along with the oral and written instructions. (See p. 330)</p> <ul style="list-style-type: none"> <li>• "Street A and Street B intersect at right angles."</li> <li>• "Street C is parallel to Street D."</li> <li>• Street F is perpendicular to streets C and D."</li> <li>• <b>Discuss</b> the terms and have students <b>translate</b> between sketch, symbolic and verbal representations. (<b>Write</b> symbolically and have students <b>translate</b> to a partner into words and draw a sketch.)</li> </ul>
<b>EXPLORE:</b> p. 331	Partners: Maps, p. 331, #14-15
<b>Practice:</b>	<ul style="list-style-type: none"> <li>• <b>ADDITIONAL EXAMPLE</b>, TE p. 330</li> <li>• <b>REVIEW TEST</b>: p. 332 (Select just a few problems.)</li> </ul>
<b>SUMMARIZE/ Closure:</b> p. 331 TE p. 331	<ul style="list-style-type: none"> <li>• <b>Select</b> students to draw their maps for p. 331; #14-17. <b>Discuss</b>.</li> <li>• <b>Write</b>: "Can two lines be both perpendicular and parallel?"</li> <li>• <b>Lesson Quiz 17.4</b>; TE p. 331</li> </ul>
<b>Homework:</b>	<ul style="list-style-type: none"> <li>• <b>CUMULATIVE REVIEW</b>: p. 333</li> <li>• <b>MIXED REVIEW AND TEST PREP</b>: p. 331; #20-22</li> </ul>

# MODULE 7: UNIT 6: GEOMETRY AND SPATIAL REASONING

## MODULE 7: DAY 4 - LESSON: 18.1

<b>MATERIALS:</b>	Plain paper, scissors, straight edge, <b>PRACTICE 18.1</b> transparency and one copy per student (optional).
<b>LESSON FOCUS:</b>	Triangles
<b>CA STANDARD:</b>	<b>MG 2.2 Key Standard</b> ; MG 2.0; (MG 1.2; MR 2.2)
<b>Purpose of Lesson:</b> Math Background: TE p. 336A	At the end of the lesson students will be able to classify triangles and solve problems involving the angle measure of triangles.
<b>Warm-up/ Routine(s):</b> <u>Books Closed</u> p. 336	<b>Mental Math: QUICK REVIEW:</b> p. 336 (On the board or overhead.)
<b>LAUNCH:</b> <u>Books Closed</u>  p. 336; Math LAB (slightly modified)  <i>The students will use this generalization to find the number of degrees in an unknown angle of a triangle; on Day 5, they will use it to determine the measure of the angles in other polygons.</i>	<p><b>Sum of the Interior Angles of a Triangle:</b> p. 336 <b>MATH LAB Activity</b></p> <ul style="list-style-type: none"> <li>Each pair of students <b>constructs</b> and cuts out one right, one obtuse, and one acute triangle. (<i>Discuss</i> each classification and have students explain to a partner how they know their triangles are classified correctly.)</li> <li>Students tear the angles off one triangle and put the angles together to find the sum. They repeat the process with the other two triangles and <b>compare</b> the sum of each of the triangles in their group.</li> <li><b>Discuss</b> straight angles and generalization.</li> <li>Have students <b>list</b> what they know about angle measurement from this lesson and the previous lessons (supplementary, complementary, vertical).</li> </ul> <p><u>To scaffold before exploration, sketch the two triangles with the common vertex at the top of page 337, and solve together; or stop and scaffold during the exploration only if it is necessary to keep them engaged in the task.</u></p>
<b>EXPLORE:</b> <u>Books Closed</u>  TE p. 337 or p. 338	<p><b>"Use what you know about vertical, complementary, and supplementary angles and the sum of the angles in a triangle to find the missing angles in these problems":</b></p> <ul style="list-style-type: none"> <li><b>Problems #4-6 and #7-12</b> from Practice 18.1; TE p. 337 <u>OR</u></li> <li>Choose from problems on p. 338; <b>#6-11 and #8-27.</b></li> </ul> <p><b>Questions to Scaffold:</b></p> <ul style="list-style-type: none"> <li>"How can you use what you know about the sum of the angles in a triangle to find the third angle?"</li> <li>"What do you know about vertical angles that might help you here?"</li> </ul>
<b>Practice:</b>	<ul style="list-style-type: none"> <li><b>Continue work from Explore.</b></li> </ul> <p>It may be helpful to first <b>discuss</b> some strategies students used to solve problems, especially ones involving the vertical and supplementary angles.</p>

<p><b>SUMMARIZE/ Closure:</b></p> <p>TE p. 339</p>	<ul style="list-style-type: none"> <li>• Students <i>sketch</i> an <u>acute, right and obtuse triangle</u> and <i>explain</i> to a partner the characteristics of each. <i>Discuss</i> as a group.</li> <li>• <i>Discuss</i> reasoning used to solve one of the missing angle problems that is more complex. <i>Represent</i> student thinking algebraically.</li> <li>• <b>Lesson Quiz 18.1, #1, and #4-6: TE p. 339</b></li> </ul>
<p><b>Homework:</b></p>	<ul style="list-style-type: none"> <li>• <b>MIXED REVIEW AND TEST PREP: p. 339</b></li> <li>• <b>p. 337 and 338: #1,5, and 28</b></li> </ul>

# MODULE 7: UNIT 6: GEOMETRY AND SPATIAL REASONING

## MODULE 7: DAY 5 - LESSON: 18.1/ 18.4

<b>MATERIALS:</b> TR59/60 CHALLENGE 18.1	TR59/60 (Run off half of the sheet of each on the same piece of paper to allow for construction of all kinds of triangles) in plastic "white board"; 4 or 5 copies of <b>CHALLENGE 18.1</b> .
<b>LESSON FOCUS:</b>	Triangles
<b>CA STANDARD:</b>	<b>MG 2.2 Key Standard; MG 2.0; MG 2.3 (MG 1.0; MR 3.2; MR 3.3)</b>
<b>Purpose of Lesson:</b> Math Background: TE p.336A	At the end of the lesson students will be able to classify triangles and solve problems involving the angle measure of triangles.
<b>Warm-up/ Routine(s):</b>	<b>Mental Math String:</b> $2 \times 180$ ; $3 \times 180$ ; $4 \times 180$ ; $5 \times 180$ ; $10 \times 180$ . <i>Discuss</i> and <i>record</i> student strategies numerically.
<b>LAUNCH:</b>  <u>Books Closed</u>  Reference p. 342  See notes at beginning of module: Harcourt defines isosceles triangles inaccurately as having exactly 2 equal sides instead of at least 2 equal sides.	<b>Isosceles, equilateral, and scalene triangles:</b> <ul style="list-style-type: none"> <li>• Using square and isometric dot paper (in plastic if you don't want to consume paper), students <i>draw</i> three triangles: all three sides equal, only two equal sides, and no equal sides.</li> <li>• <b>Introduce</b> slash marks through the sides to represent equal sides (See page 342) and have students <i>label</i> their triangles. Establish that both the triangles with two and three equal sides are isosceles, but that the one with three equal sides is called equilateral. (Can be compared to the relationship between squares and rectangles. All squares are rectangles, but all rectangles are not squares. All equilateral triangles are isosceles, but all isosceles triangles are not equilateral.)</li> </ul>
<b>EXPLORE:</b>	<ul style="list-style-type: none"> <li>• On the isometric/square dot paper, students each <i>draw</i> a right triangle, an acute triangle and an obtuse triangle.</li> <li>• Partners <i>justify</i> that each triangle fits the classification.</li> <li>• Partners add scalene, equilateral or isosceles (non-equilateral) to the classification of each triangle and <i>justify</i> classification to a partner.</li> <li>• Working in pairs, each group constructs the following triangles, using the symbol for right angles and slash marks to represent lengths of the sides: Isosceles acute; Scalene right; Isosceles obtuse; Equilateral</li> </ul>
<b>Practice:</b> TE p. 337	<b>Record diagrams from TE p. 337, ADDITIONAL EXAMPLES #2 and 3.</b> <ul style="list-style-type: none"> <li>• <i>Alternate starting problem for each group. Students work together to determine the measure of the angle and record the path to each solution.</i></li> <li>• <i>Select students to post their solutions on the board.</i></li> </ul>
<b>SUMMARIZE/ Closure:</b>  TE p. 337; CHALLENGE 18.1  TE p. 339  P. 338, #17	<ul style="list-style-type: none"> <li>• <b>Have students explain path to solutions for EXAMPLES #2 and/or #3.</b> <ul style="list-style-type: none"> <li>• "How did you decide which angle to find first?"</li> <li>• "Did everyone in the group find the measure of the same angle first?"</li> </ul> </li> <li>• <b>Select two or three statements from CHALLENGE 18.1 to display on board.</b> <i>Discuss justification for "sometimes", "never" or "always" with examples/reasoning. (To increase engagement, students may explain to partners before whole class discussion.)</i></li> <li>• <b>Lesson Quiz 18.1: #3 only: Students explain path in writing.</b></li> <li>• <b>Collect for assessment piece.</b></li> </ul>
<b>Homework:</b>	<ul style="list-style-type: none"> <li>• p. 338, #17, including path to solutions.</li> <li>• Students copy one statement from CHALLENGE 18.1: <i>Include justification for "sometimes", "never" or "always" with examples/reasoning.</i></li> </ul>

# MODULE 7: UNIT 6: GEOMETRY AND SPATIAL REASONING

## MODULE 7: DAY 6 - LESSON: 25.4

<b>MATERIALS:</b>	Calculators; string; compass; ruler (TR23); a variety of cylinders that roll: cans, paper towel and toilet paper rolls, etc. one per pair; one large cylinder for teacher use (bike wheel would be good to introduce).						
<b>LESSON FOCUS:</b>	Circumference of a Circle						
<b>CA STANDARD:</b>	<b>MG 1.1 Key Standard;</b> AF 3.2; MG 1.2; (AF 3.1; MG 1.0)						
<b>Purpose of Lesson:</b> Math Background; TE p.476A	At the end of the lesson students will understand that the ratio of the diameter of a circle to its circumference is about 1 to a little more than 3.						
<b>Warm-up/ Routine(s):</b> <u>Make choices according to the needs of the class.</u>	<b>QUICK REVIEW:</b> p. 472: <i>Choose two or three of the problems for mental computation according to student needs. Discuss/record strategies. How might the problems be related to perimeter? Write the formula for finding perimeter that matches each numerical expression.</i>						
<b>LAUNCH:</b> <u>Books Closed</u>	<p>Hold up a large cylinder or bike wheel that is different from the ones the students have. Measure the diameter with a string and mark its length on the board.</p> <ul style="list-style-type: none"> <li>Establish that the part of the circle measured is the diameter.</li> <li><b>Predict</b> whether the length of one revolution, will be greater or less than the diameter.</li> <li>"What is the name of this measurement of one revolution?" (circumference)</li> <li><b>Compare</b> the circumference to the perimeter of a polygon.</li> <li>Students suggest ways to <b>determine</b> the distance when the wheel or their cylinders have gone around one full revolution.</li> <li>"What might be another way of finding the length of one revolution without rolling it?" (See if the same length of string fits around the tire once.)</li> </ul>						
<p><b>EXPLORE:</b></p> <p><u>Books Closed:</u> Students should figure out how to use the tools (string, ruler) to measure the circumference.</p> <p>Modified from MATH LAB Activity 1: p. 474</p>	<p><b>Questions to Explore:</b> <i>"What is the relationship of the diameter of a circle to its circumference? Is there a ratio, and if so, does the ratio stay the same for circles with longer or shorter diameters?"</i></p> <p><b>Students copy a chart from the board with the following headings:</b></p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Diameter</th> <th>Predicted Circumference</th> <th>Measured Circumference</th> </tr> </thead> <tbody> <tr> <td> </td> <td> </td> <td> </td> </tr> </tbody> </table> <p><b>Ratio:</b> <math>\frac{c}{d}</math> (<i>Students record the measurements in fraction form</i>)</p> <ul style="list-style-type: none"> <li><b>Measure</b> to the nearest centimeter.</li> <li>Work in pairs and trade cylinders with other tables to <b>gather as much data</b> as possible.</li> <li>Encourage students to <b>make observations</b> after measuring each cylinder and try to use any patterns to predict the circumference after measuring the diameter of a new cylinder.</li> <li>Have students from each group <b>record data</b> from a cylinder that has not been recorded yet.</li> </ul>	Diameter	Predicted Circumference	Measured Circumference			
Diameter	Predicted Circumference	Measured Circumference					

<p><b>Practice:</b></p>	<ul style="list-style-type: none"> <li>• <b>Discuss observations.</b> (About 3 times bigger should come up.) <b>Make a sketch</b> of the relationship they describe.</li> <li>• Students <b>check conjectures</b> with calculators.</li> <li>• Students <b>write</b> in approximate circumference of 4 circles. (23m, 28 cm, 38 cm and 10 cm)</li> <li>• Using the approximate ratio they have noted, students <b>estimate</b> the diameter of each circle.</li> <li>• To <b>scaffold</b>, some students might benefit from a sketch of a circumference length stretched out that is marked with the three generic diameters and a little left over to represent the relationship they have noticed.</li> </ul> $\begin{array}{c c c} \frac{c}{3} & \frac{c}{3} & \frac{c}{3} \\ \hline d & d & d \end{array}$ <p>c(ircumference) is a little more than 3d; d(iiameter) is a little less than <math>\frac{c}{3}</math>.</p>
<p><b>SUMMARIZE/ Closure:</b></p>	<ul style="list-style-type: none"> <li>• <b>Fill in the chart</b> with the estimates but without measuring and <b>discuss</b> strategies used to determine the approximate circumference, diameter or radius.</li> <li>• <b>Record</b> the strategies numerically and connect to the diagram even if the diagram was not used. <math>C \approx 21</math>; <math>d \approx \frac{21}{7} \approx 7</math></li> </ul> <p>The "equal signs" should all be "squiggly" ones to signify approximate, not equal.</p>
<p><b>Homework:</b></p>	<ul style="list-style-type: none"> <li>• <b>MIXED REVIEW AND TEST PREP:</b> p. 475; #1-4.</li> <li>• p. 473; #10</li> </ul>

# MODULE 7: UNIT 8: MEASUREMENT: Circumference

## MODULE 7: DAY 7 - LESSON: 25.4/ 25.5

<b>MATERIALS:</b>	Calculators
<b>LESSON FOCUS:</b>	Circumference of a Circle
<b>CA STANDARD:</b>	<b>MG 1.1 Key Standard; AF 3.1; MG 1.2; (AF 3.2; MG 1.0)</b>
<b>Purpose of Lesson:</b> Math Background: TE p. 476A	At the end of the lesson students will be able to find the circumference of circles using different representations for $\pi$ and connecting the formulas to the ratio between the diameter and the circumference.
<b>Warm-up/ Routine(s):</b>	<ul style="list-style-type: none"> <li>Use data from the chart on circles (from Day 6) and have students <b>record</b> the ratio as a fraction for the measurements in the last column of the chart. <b>Ask</b> them to <b>read</b> the ratio as a division problem and use calculators to find a closer estimate of the ratio.</li> <li><b>Assign</b> groups different starting problems and have them round answers to nearest hundredth.</li> </ul>
<b>LAUNCH:</b> <u>Books Closed:</u> p. 475 and Math Background: TE p. 476A  p. 466	<ul style="list-style-type: none"> <li><b>Introduce</b> the term and symbol for <math>\pi</math>. (Refer to p. 475 and p. 476A, Math Background for discussion.) Have students use calculators to verify that <math>22/7</math> is also an approximation of <math>\pi</math>.</li> <li>Have students discuss why <math>2\pi r</math> and <math>\pi d</math> are equivalent.</li> </ul> <p><b>Introduce information about the Ferris wheels. (See p. 466)</b></p> <ul style="list-style-type: none"> <li><b>Display</b> <i>Diameters and Circumferences of Big Wheels</i> chart from page 466, but leave circumference part of the chart blank.</li> </ul>
<b>EXPLORE:</b>	<p><b>Students use what they discovered about the approximate ratio between the diameter and circumference to estimate the circumference of each of the wheels in the chart.</b></p> <p><i>"This is an estimate. Will the circumference be greater or less than your estimate?" (Have students justify using the diagram from the day before of 3 diameters plus a little more.)</i></p> <p><b>After recording the estimate using 3 as the ratio, use calculators and 3.14 or 22/7 for <math>\pi</math> to find a closer estimate of the circumference of the Ferris wheels from the chart started during Launch.</b></p> <p><i>Compare to the approximations using 3 instead of <math>\pi</math>.</i></p>
<b>Practice:</b> p. 479	<p><b>p. 479; #28, 29, 30</b></p> <ul style="list-style-type: none"> <li>Students first <b>estimate</b> the answer using 3 instead of 3.14 or 22/7. Then they solve using the formula.</li> <li><b>Record</b> numerical paths using 3 and using <math>\pi</math>.</li> </ul>
<b>SUMMARIZE/ Closure:</b> p. 479	<p><b>ASSESS: DISCUSS: TE p. 479</b> <i>Students explain to partners how to find the circumference of a circle. Then, describe with the whole class.</i></p> <p><b>ASSESS: WRITE: TE p. 479.</b> <b>Lesson Quiz 25.5; TE p. 479: Select two problems for students to solve, one using fractions and one using 3.14. Finish for homework.</b></p>
<b>Homework:</b>	<p><b>p. 478; #12-17.</b> <i>Students choose two problems to solve without a calculator.</i></p> <p><b>p. 479: MIXED REVIEW AND TEST PREP</b></p>

# MODULE 7: UNIT 8: MEASUREMENT: Circumference

## MODULE 7: DAY 8 - Lesson 26.3

<b>MATERIALS:</b>	Scissors; straight-edge; grid paper with 3 circles enclosed in squares whose sides are the length of the diameter of each circle (radius 3 units, 4 units and 5 units) <i>OR</i> grid paper and compasses for each student;
<b>LESSON FOCUS:</b> Math Background: TE p. 492A	MATH LAB: Area of a Circle
<b>CA STANDARD:</b>	<b>MG 1.1 Key Standard;</b> MG 1.2; (AF 3.0; AF 3.1; AF 3.2; MR 2.4; MR 2.5; MR 2.6)
<b>Purpose of Lesson:</b>	At the end of the lesson students will be able to approximate the area of a circle using different strategies; and derive the formula for the area of a circle from models.
<b>Warm-up/ Routine(s):</b>	<b>Display the three circles on the graph paper (see materials).</b> <i>Students use mental math to estimate the circumference of each circle.</i> <i>"What is the ratio of diameter to circumference?"</i>
<b>LAUNCH:</b>	<ul style="list-style-type: none"> <li>• <b>Display</b> the circle with a radius of 3 units drawn on grid paper.</li> <li>• With the teacher, students draw two perpendicular diameters through their circles, dividing the circle and the square enclosing the circle into 4 equal parts.</li> <li>• <b>Discuss</b> the length of the line that forms the side of one of the small squares and its relationship to the circle (3 units; it is the radius of the circle).</li> <li>• <b>"If we move area from another part of the circle to fill the squares outside the circle, how many full squares do you think the area of the whole circle will fill?"</b> <i>Write</i> their estimations (more than 3 but less than 4, between 3 and 4, etc.)</li> <li>• Choose the area of the circle from one of the squares to move to other squares outside the circle area. Put X's in units that are removed from the circle to fill outside the circle in a square and ask the students to do the same, filling whole squares and approximating partial squares.</li> </ul>
<b>EXPLORE:</b>	<p><b>"How many full square units were you able to fill?"</b> (3) <b>"Do you have more or less than half of the fourth square empty?"</b> (More)</p> <p>The idea is that the circle filled 3 and a little more of the 4 squares; the dimension of the square (radius of the circle) was 3 units. (<math>\pi</math> times <math>3^2</math>).</p> <ul style="list-style-type: none"> <li>• Students <b>find the area</b> of the 3 full squares. (<math>3 \times 3 = 9</math> sq. units and <math>9 \times 3 = 27</math> sq. units, so the area is around 27 square units.)</li> <li>• Groups assign one remaining circle to each pair and test to see if the same number of squares will be filled from the area of the circle.</li> <li>• Students <b>record</b> the numerical <i>path</i> to finding the approximate area of each circle.</li> </ul>
<b>Practice:</b>	• <b>MATH LAB: p. 491:</b> Students cut out the circle with a radius of 3 cm.

<p>This lesson continues in Day 9; it may be necessary to finish this part Day 9.</p>	<ul style="list-style-type: none"> <li>• <b>Follow the directions</b> for folding, shading and cutting the circle.</li> <li>• Form a parallelogram with the pieces of the circle.</li> <li>• "What parts of the circle form the dimensions of the parallelogram?"</li> <li>• To <b>scaffold</b>, have one student in each pair reform their parallelogram into a circle and mark the part of the circle that forms the bottom dimension of the partner's parallelogram. (It is 1/2 the circumference.)</li> <li>• <b>Draw</b> the height on the parallelogram and find it on the circle.</li> <li>• Have students verbalize that they are multiplying the base times the height and the base is 1/2 the circumference and the height is the radius : <math>\frac{\pi d}{2r}</math> . Since the radius is 1/2 the diameter it is also radius times <math>\pi</math> , times radius or <math>\pi \times r \times r (\pi r^2)</math>.</li> </ul>
<p><b>SUMMARIZE/ Closure:</b></p> <p>Students need to draw and verbalize to build an understanding of the formula for area of a circle. Estimate both circumference and area using 3 for pi before computing.</p>	<ul style="list-style-type: none"> <li>• <b>Record numerical paths</b> for the approximation of the areas.</li> <li>• "What do you notice we are doing every time?" Help students <b>make connections</b> between the square root and the radius of the circle. Get generalization in both words and symbols. (Example: "Multiply the radius by the radius and then multiply that times 3.") <math>3 \times 3 = 3</math> squared (<math>3^2</math>); So <math>r \times r = r</math> squared (<math>r^2</math>).</li> <li>• <b>Ask</b> students how the two strategies are similar. (3 and a little more of the radius squares in the first method is really <math>\pi \times</math> radius squared or <math>\pi r^2</math> .)</li> <li>• Students make two generic sketches of the models for the area of a circle: A labeled parallelogram, and a circle enclosed in a square that is divided into four radius squares. <b>Post</b> the models and students can make/use them to remind them of the meaning of the formula.</li> </ul>
<p><b>Homework:</b></p>	<p>p. 491: #1-4 (<b>Estimate</b> using 3 for <math>\pi</math> for the 4 problems instead of using 3.14.) <b>Sketch</b> both the circle model and the parallelogram model and label sketches to match the numerical path to the estimate.</p>

# MODULE 7: UNIT 8: MEASUREMENT: Circumference

## MODULE 7: DAY 9 - LESSON: 26.4

<b>MATERIALS:</b>	Calculators
<b>LESSON FOCUS:</b>	Areas of Circles
<b>CA STANDARD:</b>	<b>MG 1.1 Key Standard; MG 1.2; (NS 2.0 Key Standard; NS 2.1; AF 3.1; AF 3.2; MR 1.0)</b>
<b>Purpose of Lesson:</b> Math Background; TE p. 492A	At the end of the lesson students will be able to use a formula to find the area of a circle.
<b>Warm-up/ Routine(s):</b> <u>Books Closed</u> p. 348	<i>Display</i> sketches of 3 circles, one with a radius of 5 m; one with diameter of 20m and one with radius of 1m. Mentally <i>estimate</i> the area of two circles using sketches of one or both models. Use sketch to <i>explain</i> your solution to one problem to a partner and record numerical <i>path</i> (Remind students to use square meters for area).
<b>LAUNCH:</b> Making the sketches of the models while explaining to a partner reinforces the model and the meaning of the formula. p. 492	<ul style="list-style-type: none"> <li>• <i>Discuss</i> solutions with the models and use as a way to connect to the lesson from the day before.</li> <li>• <i>Solve</i> the three warm-up problems using 3.14 for <math>\pi</math>. (Students use calculators and groups have different starting problems.) <i>Solve</i> same problem using <math>\frac{22}{7}</math> (<math>3\frac{1}{7}</math>); then compare to the answer from the calculator solution. (Emphasize that even with 3.14, answers are not exact, but are closer than using just 3.)</li> <li>• Students <i>sketch and label</i> the wrestling mat described at the top of p. 492. Use models to estimate the area using 3 for <math>\pi</math>.</li> </ul>
<b>EXPLORE:</b>	<p><i>Sketch</i> the information in the problem: A goat is tied to a pole with a chain that allows him to graze 32 feet in any direction. How many square feet of grass does the goat have to graze in? (Students work from their sketch of a circle with a radius labeled 32 feet.) <i>Scaffold</i> the sketching according to the class. Encourage students to make a model of what is happening with a string or even a pencil representing the chain. <i>Estimate and record path</i> to estimate and then find closer estimate using 3.14 for <math>\pi</math>. Round to nearest square foot. (Use calculators and record path to solution that matches model and formula.)</p>
<b>Practice:</b> Alternative: ADVANCED LEARNERS: TE p. 492B	<p><b>p. 493:</b> Partners <i>alternate explaining</i> how to solve p. 493: #3-6. Make sketches while explaining. Include explanation of estimation using 3 for <math>\pi</math> and more accurate estimation using 3.14 for <math>\pi</math>.</p> <p><b>p. 493: #15 and 16</b></p>
<b>SUMMARIZE/ Closure:</b> TE p. 493	<ul style="list-style-type: none"> <li>• <i>Discuss</i> reasoning for #15 and 16. Press for students to verbalize, use models and relate computation to meaning of formula.</li> <li>• "How would you find the circumference for #15 and 16? How does your strategy relate to what you did to solve for area?" (Divide circumference and area by 2 and by 4.)</li> <li>• <b>ASSESS: WRITE; TE p. 493.</b></li> <li>• <b>Lesson Quiz: TE p. 493; #1 and 3.</b> Include <i>path to solution</i> and sketches. (Calculators) Finish for homework.</li> </ul>
<b>HOMEWORK:</b>	<b>MIXED REVIEW AND TEST PREP; p. 493</b>

# MODULE 7: UNIT 8: MEASUREMENT: Circumference

## MODULE 7: DAY 10: - ASSESSMENT

<b>MATERIALS:</b>	
<b>LESSON FOCUS:</b>	
<b>CA STANDARD:</b>	
<b>Purpose of Lesson:</b>	
<b>Warm-up/ Routine(s):</b>	
<b>LAUNCH:</b>	
<b>EXPLORE:</b>	
<b>Practice:</b>	
<b>SUMMARIZE/ Closure:</b>	Time to reflect back on the <b>purpose</b> of the lesson, and help students make <b>meaningful connections</b> .
<b>HOMEWORK</b>	