

SAN DIEGO CITY SCHOOLS

Institute for Learning, Department of Mathematics

Instructional Module to Enhance the Teaching of

PRENTICE HALL

PRE-ALGEBRA

California Edition 2001

GRADE 7

Module 11

Right Triangles in Algebra

THE BIG IDEAS:

By the end of grade seven, students will:

- Know the properties of, and compute with, rational numbers by manipulating numbers and equations. Know and use different representations of fractional numbers (fractions, decimals, and percents) and are proficient at changing from one to another. Understand and use factoring of numerators and denominators and properties of exponents.

Note: Negative fractions are formally introduced and studied for the first time.

- Increase their facility with ratio and proportion, compute percents of increase and decrease, and compute simple and compound interest.
- Graph linear functions and understand the idea of slope and its relation to ratio. Solve simple linear equations and inequalities over the rational numbers.
- **Know the Pythagorean theorem and solve problems in which they compute the length of an unknown side.**

Note: The Pythagorean theorem is probably the first true theorem that the students will have seen.

- Know how to compute the surface area and volume of basic three-dimensional objects and understand how area and volume change with a change in scale.
- Make conversions between different units of measurement. Know and use various forms of displays for data sets.

(Notes from Mathematics Framework for California Public Schools, Grade Seven, p149-152.)

Key Mathematical Concepts Addressed:

MODULE 11: Right Triangles in Algebra (Chapter 11)

- ***MG 3.3 Key Standard - Know and understand the Pythagorean Theorem and its converse and use it to find the length of the missing side of a right triangle and the lengths of other line segments and, in some situations, empirically verify the Pythagorean theorem by direct measurement.**
- ***NS 1.4 Classify Real Numbers**
- ***NS 2.4 Find Square Roots**

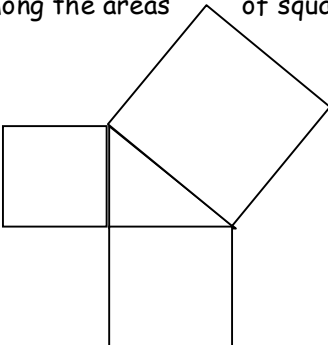
MODULE 11: Right Triangles in Algebra

DAY 1: Lesson 11 - 1; pp. 560-564

LESSON FOCUS:	To find square roots of numbers and to classify real numbers
CA STANDARD:	AF 2.2 NS 1.4 Key Standard NS 2.4
Purpose of Lesson:	At the end of the lesson, students will be able to use square roots in real-world situations (such as finding the distance to the horizon).
Warm-up/ Routine(s): Books closed: TE p. 560 Connecting to Students' Lives	p. 560 TRY THIS #1-4. Write the following on the overhead or board: <ul style="list-style-type: none"> • Sketch a square 3 x 3 units. <i>How many units are in the square?</i> • Simplify: $\sqrt{100}$; $-\sqrt{100}$; $\sqrt{16}$; $-\sqrt{16}$ • Estimate $\sqrt{17}$ to the nearest whole number. <i>Explain how you arrived at your solution.</i> Vocabulary: The inverse of squaring a number is finding a square root.
LAUNCH: p. 561; Part 2: Ex #3 a-g and #9-12; p. 562 #1-15 Reference	Ask students to write <i>all</i> characteristics of the values for each list: (Use additional or different values as appropriate for your students.) $\rightarrow \sqrt{4}; \frac{1}{8}; -\sqrt{36}; \frac{10}{5}; \sqrt{64}; 25; -7; \sqrt{121}; 432.8; 0.1212\dots$ $\rightarrow -\sqrt{3}; \sqrt{61}; 0.120120012\dots; \pi; \sqrt{2}; \sqrt{18}; -\sqrt{24}; \sqrt{10}$ *A number that cannot be expressed as the ratio of two integers $\frac{a}{b}$, where b is not zero, is irrational . If a positive integer is not a perfect square, then its square root is irrational.
EXPLORE: p. 561, Ex #2 Real-World Connection	p. 559 (For the student.) Estimate the distance to the horizon. <i>In pairs, ask the students to sketch the lifeguard's line of site (10 feet above the ground) to the horizon. While sketching, students should be discussing ways to estimate the distance to the horizon.</i> *To estimate the distance (in miles) to the horizon use the formula $d = \sqrt{1.5h}$. $(d = \sqrt{1.5 \cdot 10} = \sqrt{15}; \sqrt{9} < \sqrt{15} < \sqrt{16}; \sqrt{16}$ is the closest perfect square = 4. <i>The lifeguard can see about 4 miles to the horizon.)</i> <i>How/ Why does the formula help to estimate the distance?</i>
Practice:	p. 563; Algebra #39-42 or TE p. 561; Lesson Quiz #1-4
SUMMARIZE:	Exit Slip: How would you determine the square root of a number if it is not a perfect square? Give an example.
Closure:	Time to reflect back on the purpose of the lesson, and help students make meaningful connections.
Homework: p. 562 #37b p. 563 #48	<ul style="list-style-type: none"> • p. 562 #37b <i>Open-ended:</i> Name three rational numbers and three irrational numbers between 9 and 10. Explain how you know that each number is rational or irrational. And • p. 563; #48.

MODULE 11: Right Triangles in Algebra

DAY 2: Lesson 11- 2; pp. 564-569

LESSON FOCUS:	The Pythagorean Theorem
CA STANDARD:	MG 3.3 Key Standard
Purpose of Lesson:	At the end of the lesson, students will be able to investigate and build an understanding of the Pythagorean Theorem
Warm-up/ Routine(s):	Test-Taking Tips on Transparencies (Recognizing a pattern.) TE p. 557G
LAUNCH: Dot paper (Trans. p. 5)	On dot paper, ask students to draw squares of varying size and find the area: (Example: 2×2 ; 4 or 3×3 ; 9 , etc)
EXPLORE: Materials needed: Dot paper (Trans. p. 5) Table (Following page) Calculators (optional) 5x8 index cards CMP: Looking For Pythagoras; pp. 27-28 Reference	<p>Investigating the Pythagorean Theorem: Students will look for a relationship among the areas of squares drawn on the sides of right triangles.</p>  <p>Using the table provided:</p> <ul style="list-style-type: none"> • (For each row) Students draw a right triangle with the given lengths on dot paper. Then draw a square on each side of the triangle. • Students find the areas of the squares on the legs and on the hypotenuse for each triangle. They record their results in the (provided) table. • Ask students to look for a pattern in the relationship among the areas of the three squares drawn for each triangle. Use the pattern to make a conjecture about the relationship among the areas. • Draw a right triangle with side lengths that are different from those given in the table. Use this triangle to test the conjecture from part C.
Practice:	Add another column to the table. In the column, record the hypotenuse length of each triangle using the square root ($\sqrt{\quad}$) symbol.
SUMMARIZE:	Table Talk: Describe the patterns that you see in the table. Share out as a class.
Closure:	Time to reflect back on the purpose of the lesson, and help students make meaningful connections.
Homework:	<ol style="list-style-type: none"> 1) Do you think that the Pythagorean Theorem will work for triangles that are not right triangles? Explain* 2) Do you think that the Pythagorean Theorem will work for all right triangles? Explain* <p>*Students should be encouraged to use drawings, words or symbols to explain and justify their thinking.</p>

Investigating the Pythagorean Theorem:

Students look for a relationship among the areas of squares drawn on the sides of right triangles.

- Each group given 1-2 (time permitting) different triangles.
- Draw the right triangle with the given lengths on dot paper. Then draw a square on each side of the triangle. Use a 5x8 card as a right angle tool to construct the square on the hypotenuse.
- Students find the areas of the squares on the legs and on the hypotenuse for each triangle. (Rotate the hypotenuse square at the vertex for ease of finding the area as noted on the diagram on the previous page.) Record results in the table.
- Look for a pattern in the relationship among the areas of the three squares drawn for each triangle. Use the pattern to make a conjecture about the relationship among the areas. Class debrief.
- Draw a right triangle with side lengths that are different from those given in the table. Use this triangle to test the conjecture.

Length of leg 1	Length of leg 2	Area of square on leg 1	Area of square on leg 2	Area of square on hypotenuse
1	1	1	1	2
1	2			
2	2			
1	3			
2	3			
3	3			
3	4			

- Add another column to the table. In the column, record the hypotenuse length of each triangle using the square root ($\sqrt{\quad}$) symbol.

HOMEWORK:

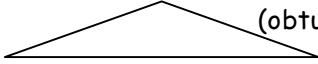

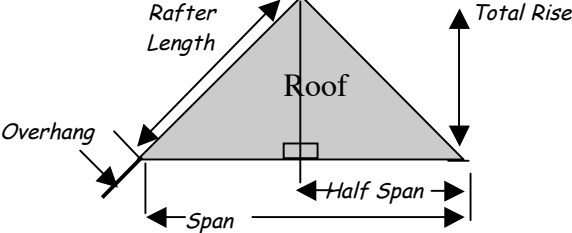
- 1) Do you think that the Pythagorean Theorem will work for triangles that are not right triangles? Explain*

- 2) Do you think that the Pythagorean Theorem will work for all right triangles? Explain*

*Use drawings, words or symbols to explain and justify their thinking.

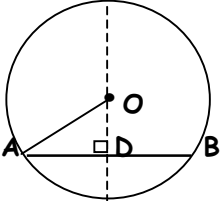
MODULE 11: Right Triangles in Algebra

DAY 3: Lesson 11- 2; pp. 564-569

LESSON FOCUS:	To identify right triangles and to use the Pythagorean Theorem
CA STANDARD:	MG 3.3 Key Standard NS 2.4
Purpose of Lesson:	At the end of the lesson, students will be able to use the Pythagorean Theorem in real-world situations, such as carpentry.
Warm-up/ Routine(s):	<p>TE p. 562; Challenge:</p> <ul style="list-style-type: none"> Classify the following triangles by its angles: <div style="display: flex; justify-content: space-around; align-items: center; margin: 10px 0;">   </div> Does $\sqrt{36} = \sqrt{9} \cdot \sqrt{4}$? (yes) Explain Does $\sqrt{25} = \sqrt{9} + \sqrt{16}$? (no) Explain
LAUNCH: TE pp. 564-565 Lesson Background & Connecting to History; p. 565; <i>Introducing the Concept</i>	<ul style="list-style-type: none"> Refer to work from Day 2. Ask students what pattern(s) they discovered among the areas of the three squares drawn for each triangle. What conjecture about the relationships did they make? p. 565; <i>Introducing the Concept</i> #1 and #2: Will the student's conjectures work for the c^2 value in the table?
EXPLORE: Books closed: p. 566; Ex #3 Calculators	<p>(Connect back to Chapter 10-2; p. 510)</p> <ul style="list-style-type: none"> p. 566; Example 3: Measurement Connection (Reference) <p>Carpentry terms: <i>span, rise, rafter length</i></p> <div style="text-align: center;">  </div> <p><i>How else might a carpenter use geometry in their work?</i></p>
Practice:	p. 568; Algebra #33-36
SUMMARIZE:	<p>Have student groups share their results from the Explore situation and their understandings of how to use the Pythagorean Theorem. Push student thinking to make connections with what they did yesterday (or previously in the Investigation Unit).</p>
Closure:	Time to reflect back on the purpose of the lesson, and help students make meaningful connections.
Homework: p. 566; TRY THIS #6	<ul style="list-style-type: none"> p. 566; TRY THIS #6: Draw and label a roofline with the following lengths. What is the rise of a roof if the span is 22 feet and rafter length is 14 feet? Round to nearest tenth of a foot.

MODULE 11: Right Triangles in Algebra

DAY 4: Lesson 11- 2A; pp. 570-571

LESSON FOCUS:	The Pythagorean Theorem and Circles
CA STANDARD:	MG 3.3 Key Standard
Purpose of Lesson:	At the end of the lesson, students will be able to use the Pythagorean Theorem to find lengths of chords and radii of circles.
Warm-up/ Routine(s): TE p. 516; Math Reasoning	TE p. 516; Math Reasoning: <i>Which has a larger area, a circle with a radius of 2 or a square with a side of 4? Prove your solution with a sketch and a statement.</i>
LAUNCH: TE p.570; Math Background	<p>p. 570; The Pythagorean Theorem and Circles</p>  <p>Students construct the circle on p. 570; following steps 1-4. • <i>Classify $\triangle AOD$ by its angles.</i></p>
EXPLORE:	<p>With a partner or in small groups ask students to write a conjecture about the perpendicular bisector of a chord and the center of the circle.</p> <p>Reference <u>Circle Folding Investigation</u> as source for building needed vocabulary/concepts. (Found in Module 9, Appendix)</p>
Practice:	p. 571; #3 and #6
SUMMARIZE:	Share out as a class. Teacher charts conjectures/thinking. Make connections with previous experiences.
Closure:	Time to reflect back on the purpose of the lesson, and help students make meaningful connections.
Homework:	p. 571; #4-5 and 10-11

MODULE 11: Right Triangles in Algebra

DAY 5: Lesson 11- 3; pp. 572-576

LESSON FOCUS:	Distance Formula
CA STANDARD:	MG 3.3 Key Standard NS 2.4
Purpose of Lesson:	At the end of the lesson, students will be able to find the distance between two points using the Distance Formula and find the perimeters of figures on the coordinate plane.
Warm-up/ Routine(s):	TE p. 572 Daily Skills Warm-Up Transparency (Even or odd problems)
LAUNCH: p. 572 Materials needed: Grid paper & Rulers	On grid paper, have students make a coordinate grid -5 to 10, in 1 unit increments. Next, have students mark point A at (2,1) and point B at (7,3). Have students complete a triangle by placing point C at (7,1). Using the formula $(AB)^2 = (AC)^2 + (BC)^2$ (Where AC equals the difference in x-values; BC equals the difference in y-values)
EXPLORE: Materials needed: Grid paper & Rulers Calculators	On the same coordinate grid the students complete a triangle by placing point C at (2,3). <i>Ask students to explain why or why not this placement of C will make a difference in the solution.</i> Students should test and prove their conjecture.
Practice:	p. 573; TRY THIS #1 and #2. On graph paper, draw the points to form a right triangle. Find the distance between each pair of points.
SUMMARIZE:	Partner Talk/Write: How is the Pythagorean Theorem or the Distance Formula used to find the perimeter of figures on the coordinate plane?
Closure:	Time to reflect back on the purpose of the lesson, and help students make meaningful connections.
Homework: TE p. 576	p. 575; Check Understanding #1-3 or Reteaching 11-3

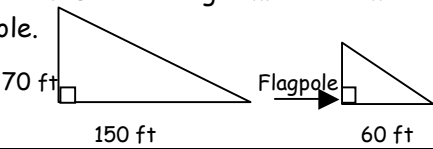
MODULE 11: Right Triangles in Algebra

DAY 6: Lesson 11- 3; pp. 572-576

LESSON FOCUS:	Midpoint Formula
CA STANDARD:	MG 3.3 Key Standard NS 2.4
Purpose of Lesson:	At the end of the lesson, students will be able to find the midpoint of a segment using the Midpoint Formula, to find the perimeters of figures on the coordinate plane.
Warm-up/ Routine(s): Alternative Assessment: Chapter Support File p. 27	Alternative Assessment: Chapter Support File p. 27; #2. <i>Draw a right triangle with legs 5 in. and 12 in. Find the length of the hypotenuse.</i>
LAUNCH: p. 573 Materials needed: Grid paper & Rulers	Ask students to think of other words to express the term <i>midpoint</i> . (1 min) On grid paper, have students make a coordinate grid -10 to 10, in 1 unit increments. Next, have students draw \overline{AB} from (-3,2) to (7,-2). Have students estimate the midpoint of the line segment.
EXPLORE: p. 574, PART 2	Using the Midpoint Formula ask students if their estimate is the same as using the formula? Test with p. 574; TRY THIS #4 and #5.
Practice:	p. 574; TRY THIS #4 and #5
SUMMARIZE:	<u>QUICKWRITE</u> : Explain the Midpoint Formula to a student in your class who was absent today. Explain <i>why</i> are there two numbers (ordered pair) in the solution.
Closure:	Time to reflect back on the purpose of the lesson, and help students make meaningful connections.
Homework:	p. 575; Check Understanding: #4-6

MODULE 11: Right Triangles in Algebra

DAY 7: Lesson 11- 4; pp.578-581

LESSON FOCUS:	Reasoning Strategy: Write a Proportion
CA STANDARD:	AF 4.2 Key Standard MR 3.2
Purpose of Lesson:	At the end of the lesson, students will be able to measure distances indirectly.
Warm-up/ Routine(s): p. 290; Ex 2; TRY THIS #2	A building 70 feet high casts a 150-ft shadow. A nearby flagpole casts a 60-ft shadow. Draw a diagram. Use similar triangles to find the height of the flagpole.  $\frac{70}{150} = \frac{f}{60} \quad (f=28\text{ft})$ <p>The height of the flagpole is 28 ft</p>
LAUNCH: TE p. 578: Connecting to Students' Lives TE p. 579 Connecting to Geography ALSO: 11.1; pp. 560-561	TE p. 578: Connecting to Students' Lives; TE p. 579 Connecting to Geography: Ask students to discuss possible ways for finding measurements without actually taking them; such as for a tall building, a tree, or a mountain. Brainstorm by groups. Connect back to the lifeguard measuring the distance to the horizon in 11.1.
EXPLORE: Books closed: p. 578; Surveying the Grand Canyon Calculators	p. 578; Surveying the Grand Canyon Students work in partnerships; collaborate with table group.
Practice: TE p. 578-579; Additional Ex #2	TE p. 578-579; Additional Example #2: Ask students to draw a diagram and find the height of the building. <i>At a given time of day, a building of unknown height casts a shadow that is 24 feet long. At the same time of day, a lamp post that is 8 feet tall cast a shadow that is 4 feet long.</i>
SUMMARIZE:	Share out strategies/findings with the whole class.
Closure:	Time to reflect back on the purpose of the lesson, and help students make meaningful connections.
Homework:	Complete Practice Problem <i>or</i> p. 581 #8, 9 and 10

MODULE 11: Right Triangles in Algebra

DAY 8: Lesson 11- 5; pp.582-586

LESSON FOCUS:	Special Right Triangles (45° - 45° - 90° and 30° - 60° - 90°)
CA STANDARD:	MG 3.3 Key Standard
Purpose of Lesson:	At the end of the lesson, students will be able to find distances in real-world situations
Warm-up/ Routine(s): p. 581; #12	p. 581; #12: <i>#12: Christa thought of a number. She added 4, multiplied the sum by -5, and subtracted 12. She then doubled the result and got -34. What number did Christa start with? (-3)</i>
LAUNCH: Books closed: p, 583; Example 2: TRY THIS #2	p, 583; Example 2: TRY THIS #2 (Books closed) Gymnasts use floor mats that are 12 m by 12 m for floor exercises. A gymnast does cartwheels across the diagonal mat. What is the length of the diagonal to the nearest meter and why might the gymnast do cartwheels across the diagonal rather than straight across the mat? (17 meters; the diagonal is longer than the sides, giving more room & safer.)
EXPLORE: TE p. 583; Ex #2	Exploring distances on baseball diamonds using the Pythagorean Theorem and the relationships in special right triangles.
Practice: Additional Ex #1-2; p. 582	Additional Example #1-2; p. 582
SUMMARIZE:	<u>QUICKWRITE:</u> <i>Have students reflect on where in the real-world special right triangles are found. What makes a right triangle "special"?</i>
Closure:	Time to reflect back on the purpose of the lesson, and help students make meaningful connections.
Homework:	Complete Additional Example #1-2; or TE p. 582; Connecting to Students' Lives with origami.

MODULE 11: Right Triangles in Algebra

DAY 9: Assessment

LESSON FOCUS:	Right Triangles in Algebra
CA STANDARD:	Standards addressed in Chapter 11.
Purpose of Lesson:	To assess the students' understanding of the key concepts and skills found in Chapter 9.
Warm-up/ Routine(s): p. 581; #11	#11: <i>Hai takes 12 minutes to walk to school. He wants to arrive 15 minutes early to meet his friends. What time should he leave his house if school begins at 8:10am? (7:43am)</i>
LAUNCH:	Assessment- teacher selected
EXPLORE:	
Practice:	
SUMMARIZE:	<u>Reflective Writing:</u> In their math journals, have students reflect upon what they consider to be the key learnings - the BIG IDEAS - from this chapter. Invite them to ask any questions that they may have related to this unit of study.
Closure:	Time to reflect back on the purpose of the lesson, and help students make meaningful connections.
Homework:	

MODULE 11: Right Triangles in Algebra

DAY 10: Lesson 11- 6; pp.588-592

Optional

LESSON FOCUS:	Sine, Cosine, and Tangent Ratios
CA STANDARD:	MR 1.1
Purpose of Lesson:	At the end of the lesson, students will be able to find and use trigonometric ratios in right triangles
Warm-up/ Routine(s):	
LAUNCH:	
EXPLORE:	
Practice:	
SUMMARIZE:	
Closure:	Time to reflect back on the purpose of the lesson, and help students make meaningful connections.
Homework:	

MODULE 11: Right Triangles in Algebra

DAY 11: Lesson 11- 7; pp.594-598

Optional

LESSON FOCUS:	Angles of Elevation and Depression
CA STANDARD:	MR 2.2
Purpose of Lesson:	At the end of the lesson, student will be able to use trigonometry for finding angles of elevation and angles of depression to solve real-world problems such as in surveying and navigation.
Warm-up/ Routine(s):	
LAUNCH:	
EXPLORE:	
Practice:	
SUMMARIZE:	
Closure:	Time to reflect back on the purpose of the lesson, and help students make meaningful connections.
Homework:	