

PLAYING
WITH THE
PUZZLE OF
PYTHAGORAS

GRADE 7

**A Six Day Hands-On
Investigative Unit of Study
(Foundational for Prentice Hall
Pre-Algebra, Chapter 11)**

(Timeline: To be implemented following Winter Break)

KEY MATHEMATICAL CONCEPTS FOR GRADE SEVEN

THE BIG IDEAS:

By the end of grade seven, students will:

- Know the properties of, and compute with, rational numbers by manipulating numbers and equations. Know and use different representations of fractional numbers (fractions, decimals, and percents) and are proficient at changing from one to another. Understand and use factoring of numerators and denominators and properties of exponents.
Note: Negative fractions are formally introduced and studied for the first time.
- Increase their facility with ratio and proportion, compute percents of increase and decrease, and compute simple and compound interest.
- Graph linear functions and understand the idea of slope and its relation to ratio. Solve simple linear equations and inequalities over the rational numbers.
- **Know the Pythagorean theorem and solve problems in which they compute the length of an unknown side.**
Note: The Pythagorean theorem is probably the first true theorem that the students will have seen.
- Know how to compute the surface area and volume of basic three-dimensional objects and understand how area and volume change with a change in scale.
- Make conversions between different units of measurement. Know and use various forms of displays for data sets.

(Notes from Mathematics Framework for California Public Schools, Grade Seven, p149-152.)

Key Mathematical Concepts Addressed:

Pythagorean Theorem Investigation

- ***MG 3.3 Key Standard - Know and understand the Pythagorean Theorem and its converse and use it to find the length of the missing side of a right triangle and the lengths of other line segments and, in some situations, empirically verify the Pythagorean theorem by direct measurement.**

DAY 1: Pythagorean Theorem Investigation

LESSON FOCUS:	To build foundational concepts to empirically verify Pythagorean Theorem
CA STANDARD:	*MG 3.3 KEY STANDARD
Purpose of Lesson:	At the end of the lesson students will be able to form an array to find the area of a rectangle.
Warm-up/	Develop Number Sense: Measurement - Capacity 1 cup = 8 fluid ounces (oz) 1 pint = 2 cups 1 quart = 2 pints 1 gallon = 4 quarts <i>What would be the best measure to use for the following:</i> <i>Amount of: A) water in a swimming B) coffee in a mug C) liquid in an eyedropper D) cream needed to make ice cream E) school milk carton</i>
LAUNCH:	<i>BRIEF TABLE TALK: What do we know about rectangles and their properties?</i> <i>List three things that your group is certain of & give an example for each. Share out with class. Chart responses.</i> (Formula for area of a rectangle may or may not arise but that is not the point of the lesson - students will have further opportunities to explore & extend this in Module 10 later in the school year.)
EXPLORE: Materials: pattern block squares or color tiles, cm graph paper	Investigating Area of a Rectangle: Build Arrays Have students work together at tables to use pattern block squares or square tiles to form arrays to find area of a variety of rectangles. <i>Build a rectangle with sides 3 tiles and 4 tiles; 2 tiles and 6 tiles; 1 X 8; 4 X 4 . . . Each time record the length & the width and how many tiles it takes to make the complete rectangle (area).</i> (May use geoboards, if available.) (If manipulatives are NOT available, have them draw rectangles on grid paper & find area. Share out strategies.) <i>If you have 12 tiles, how many different rectangles can you make? List them by length & width. 16 tiles? 20 tiles? 24 tiles? 25 tiles?</i>
Practice: Vocabulary: Square number, square root	Find the number of possible rectangles with 36 tiles. <i>What do you notice about the rectangles made from 16, 25 and 36 tiles?</i> (Discuss square numbers (n^2) - how to find a square - square root - should be review.)
SUMMARIZE:	QUICK WRITE: <i>Explain how you can use tiles to find the area of a rectangle with sides of 4 tiles and 7 tiles. Explain another way to find area of a 4 X 7 rectangle?</i>
Closure:	Time to reflect back on the purpose of the lesson, and help students make meaningful connections.
Homework:	Complete <u>Practice</u> & find the factors of 12, 16, 20, 24 & 25. What do the factors have in common with the rectangle arrays you built?

DAY 2: Pythagorean Theorem Investigation

LESSON FOCUS:	To build foundational concepts to empirically verify Pythagorean Theorem
CA STANDARD:	*MG 3.3 KEY STANDARD
Purpose of Lesson:	At the end of the lesson, students will understand the relationship between a right triangle & a rectangle and will be able to find the area of a right triangle.
Warm-up/	Develop Number Sense: Review Prime & Composite <i>Is your age a prime number or a composite number?</i> <i>List the numbers from 10 to 20 that are prime. (11, 13, 17, 19)</i> <i>List those from 10-20 that are composite. (10,12,14,15,16,18,20)</i> <i>Are there any square numbers between 10 & 20? (16)</i>
LAUNCH:	TABLE TALK: <i>What do we know about triangles and their properties?</i> <i>List three things that your group is certain of & give an example for each.</i> Share out with class. Chart responses.
EXPLORE: Materials: Cm graph paper, ruler, scissors	<i>What do rectangles and triangles have in common? Differences?</i> <u>Investigate finding area of a triangle:</u> Use graph paper to draw a rectangular array similar to those built on Day 1. Record the length & width and the area of the rectangle. Model how to draw a diagonal line from one vertex to the opposite vertex of the rectangle. Cut along the diagonal to form two equal right triangles. With students help, count the number of cm squares in a triangle. Discuss strategies for dealing with parts of a square. Record finding. Have students work with a partner to solve for area of a number of triangles formed from rectangles, i.e. 3X4, 4X5, 6X8, 4X4, . . . <i>What do you notice about the two triangles & the rectangle? About the area of the rectangle & the area of one of the triangles formed from it?</i>
Practice:	Have students form rectangles of their own dimensions & solve for area of triangles.
SUMMARIZE:	<u>Exit Slip:</u> <i>You have a right triangle cut from a rectangle with area of 18 square units, what would the area of the rectangle be? How do you know? Do you know what the lengths of the sides are? Why or why not?</i>
Closure:	Time to reflect back on the purpose of the lesson, and help students make meaningful connections.
Homework:	<i>What is the relationship between a rectangle and a right triangle formed from it? Explain two ways to find the area of the right triangle & give an example for each.</i>

DAY 3: Pythagorean Theorem Investigation

LESSON FOCUS:	To build foundational concepts to empirically verify Pythagorean Theorem
CA STANDARD:	*MG 3.3 KEY STANDARD
Purpose of Lesson:	At the end of the lesson, students will begin to understand the relationship between a right triangle & a rectangle and will be able to use this understanding to find the area of a right triangle.
Warm-up/	Develop Number Sense: Review - Gather/Organize Data <i>In December 1925, A Children's Story by A.A. Milne was published. This started the Winnie-the-Pooh series. Pooh said that what he liked best in all the world was eating honey. What do you like best in the world? What do you wish you never had to do again? Take a poll in your class. Organize the results. Are there any "outliers"? Find the mode.</i>
LAUNCH: TE p564	Share TE p564, <i>Background for the Lesson</i> with students. Have students share any additional information they may have about Pythagoras.
EXPLORE: Materials: Metric rulers, p. 1 & 2 Pythag. Investigation (1/pair), calculators available Vocabulary: Legs, hypotenuse	<u>Investigating the sides of right triangles:</u> Students work with a partner to measure each triangle & fill in the table on p1. (See following pages.) (You may need to clarify what a^2, b^2, c^2 means as far as computation.) Partners engage in conversation at their table group about their computational results.
Practice:	Measure the sides of some of the triangles formed Day 2 & record data similar to the table, p 1, Investigation.
SUMMARIZE:	TABLE TALK/WHOLE CLASS: <i>What observations were you able to make from studying your results in the right side of the table?</i>
Closure:	Time to reflect back on the purpose of the lesson, and help students make meaningful connections.
Homework:	Square the numbers 5-12. Find square roots for 25, 36, 64, 100, 169, 225. <i>What operation do you use to square a number? To find square root?</i>

DAY 4: Pythagorean Theorem Investigation

LESSON FOCUS:	To build foundational concepts to empirically verify Pythagorean Theorem
CA STANDARD:	*MG 3.3 KEY STANDARD
Purpose of Lesson:	At the end of the lesson, students will experience & understand that the sum of the areas made from the legs of the right triangle are equal to the area of the square made from the hypotenuse.
Warm-up/	Develop Number Sense: <i>Aaron has read the first 60 pages of a book. When he has read 35 more pages, he will have read half the book. How many pages are in the book? How do you know? (What's the title of the book?)</i>
LAUNCH: TE p565	Share TE p565, <i>Connecting to History</i> with students. Reflect upon observations from Day 3 Investigation.
EXPLORE: Materials: p. 3 Pythag. Investigation (1/pair), Cuisenaire rods or cm graph paper Vocabulary: Legs, hypotenuse, area	<u>Investigating area of squares made from sides of right triangles:</u> Students work with a partner to cover squares 1 & 2 on p3 (See following pages.) with Cuisenaire rods and fit them into square 3 (exchanges may be made). IF YOU DO NOT HAVE CUISENAIRE RODS AVAILABLE TO YOU, STUDENTS CAN USE CM GRAPH PAPER TO MAKE WHAT THEY NEED FOR THIS INVESTIGATION. Table groups discuss their strategies for solving the problem and their observations.
Practice:	<i>Draw another right triangle of your own dimensions. Construct squares from each side and investigate with the rods. What do you find?</i>
SUMMARIZE:	PARTNER TALK/WRITE: <i>If I know the area of the squares made from the legs of a right triangle, do I know anything about the area of the square made from the hypotenuse? Explain your excellent Pythagorean thinking!</i>
Closure:	Time to reflect back on the purpose of the lesson, and help students make meaningful connections.
Homework:	Review simplifying fractions, p217 #26-37, Account for all possibilities, p217 #38

DAY 5: Pythagorean Theorem Investigation

LESSON FOCUS:	To build foundational concepts to empirically verify Pythagorean Theorem
CA STANDARD:	*MG 3.3 KEY STANDARD
Purpose of Lesson:	At the end of the lesson, students will experience & understand that the sum of the areas made from the legs of the right triangle are equal to the area of the square made from the hypotenuse.
Warm-up/	Develop Number Sense: Measurement <i>The sound of thunder travels about one mile in five seconds. Suppose a bolt of lightning strikes 3 miles away from you, how long does it take for the sound of thunder to reach you? If it took about a half a minute for the sound to reach you, about how far away would the lightning have struck?</i>
LAUNCH:	Share some of the responses from the PARTNER TALK/WRITES from yesterday. Class discussion - <u>make connections</u> .
EXPLORE: Materials: p. 4 Pythag. Investigation (1/pair), rulers, scissors Vocabulary: Legs, hypotenuse, area, midpoint	<u>Investigating right triangles:</u> Students work with a partner cutting squares made from the legs of a right triangle, p4, (See following page.) and fitting the pieces into the area of the square made from the hypotenuse of the right triangle. **Model how to find a midpoint. Table groups discuss their strategies for solving the problem and their observations.
Practice:	<i>Construct another right triangle of your own dimensions & form squares from the sides. Repeat this investigation on your triangle. Did you find the same results?</i>
SUMMARIZE:	QUICK WRITE: <i>How is this investigation related to the investigation with the Cuisenaire rods? Explain your excellent Pythagorean thinking!</i>
Closure:	Time to reflect back on the purpose of the lesson, and help students make meaningful connections.
Homework:	Review solving equations: P 91, #31-42 (Challenging) OR p 91, 15-30

DAY 6: Pythagorean Theorem Investigation

LESSON FOCUS:	To build foundational concepts to empirically verify Pythagorean Theorem
CA STANDARD:	*MG 3.3 KEY STANDARD
Purpose of Lesson:	At the end of the lesson, students will understand when they might use the Pythagorean Theorem & be able to use it to solve real life problems.
Warm-up/	Develop Number Sense: <i>A student has eight shirts and six pairs of pants. How many different shirts and pants outfits can he choose? What strategy did you use? Why? Check with your neighbor & see if he/she has the same result. Did they use the same or a different strategy to solve the problem?</i>
LAUNCH: p 564, PART 1 p 565, Ex. 1	Students turn to p 564 and read together PART 1, <i>Using the Pythagorean Theorem</i> . This is what they have been investigating in a more concrete fashion. Now they can use what they know to find a missing side of a right triangle. Do p565, EXAMPLE 1 together. Do TRY THIS, # 3 together.
EXPLORE: <u>Materials:</u> p. 5 Pythag. Investigation (1/pair)	Students work with a partner to solve real world problems using the Pythagorean Theorem. Investigation p 5, #1 and #2 (See following page.). Share their results with the students at their table.
Practice:	Write a story problem that would have you use the Pythagorean Theorem to solve it! Find your solution.
SUMMARIZE:	REFLECTION: <i>In your brilliant Pythagorean thinking, tell what you learned about right triangles. (You might want to draw an example.) How could you use this knowledge to solve a real world problem?</i>
Closure:	Time to reflect back on the purpose of the lesson, and help students make meaningful connections.
Homework:	<i>A rectangular park measures 300 ft by 400 ft. A sidewalk runs diagonally from one corner to the opposite corner. Find the length of the sidewalk. How do you know? Explain your thinking.</i>