Grade 3

Concepts of Equality

Purpose:

- To develop an appropriate conception of equality and the equal sign
- To develop understanding that the equal sign denotes the relation between two equal quantities (rather than a command to carry out a calculation)

Description:

- Students are engaged in a discussion about the meaning of the equal sign.
- The context of this discussion is true/false and open number sentences.
- The number sentences provide a focus for students to articulate their ideas and to challenge their conceptions.
- The discussions assist in developing ways of thinking and communicating that embody the principles of algebraic reasoning.
- Students articulate mathematical principles that often are not explored or stated.
- Students must justify the principles that they propose in ways that convince others, and they must recognize and resolve conflicting assumptions and conclusions,

Materials:

Purposely planned number sentences and open number sentences. The numbers selected should be easily accessible to students. The focus is on the meaning of the equal sign, not on practice of operations.

Time: 15 minutes maximum

Pre-assessment

Before beginning this series of routines, ask your students to complete the following on a half-sheet of paper:

What number would you put in the box to make this a true number sentence?

$$8 + 4 = \square + 5$$

This information is for you. Do not discuss this problem with your students.

This problem was given to thirty typical elementary-grade classes. The responses were as follows:

	Response - Percent Responding			
	7	12	17	12 and 17
Grades 1 and 2	5%	58%	13%	8%
Grades 3 and 4	9%	49%	25%	10%
Grades 5 and 6	2%	76%	21%	2%

From: Falkner, Levi, & Carpenter, 1999

This data suggests that many elementary school students have serious misconceptions about the meaning of the equal sign as a relation between two equal quantities. Many seem to interpret the equal sign as a command to carry out a calculation (the answer is…).

This misconception limits students' ability to learn basic arithmetic ideas with understanding and their flexibility in representing and using those ideas. This creates even more serious problems as they move to algebra.

Directions:

1. Engage students in a general discussion about true/false number sentences or what it means for a number sentence to be true or false. Provide examples asking whether the number sentence is true or false and how they know it is true or false. For example:

   8 – 5 = 3 2 x 2 = 5 99 + 68 = 167

2. Once students are familiar with true/false number sentences, equations can be introduced that may encourage them to examine their conceptions of the meaning of the equal sign. Pose one equation at a time and lead a discussion as to whether the equation is true or false. Students must justify their claims. Do not tell. Lead a discussion and ask questions.

   For example:

   4 + 5 = 9 9 = 4 + 5 9 = 9
   4 + 5 = 4 + 5 4 + 5 = 5 + 4 4 + 5 = 6 + 3

   2 x 4 = 8 8 = 2 x 4 8 = 8
   2 x 4 = 2 x 4 2 x 4 = 4 x 2 2 x 4 = 1 x 8

   15 – 7 = 8 8 = 15 – 7 8 = 8
   15 – 7 = 15 – 7 15 – 7 = 7 - 15 15 – 7 = 16 – 8

10 ÷ 2 = 5	5 = 10 ÷ 2	5 = 5
10 ÷ 2 = 10 ÷ 2	10 ÷ 2 = 2 ÷ 10	10 ÷ 2 = 15 ÷ 3

Many of the examples above do not follow the familiar form with two numbers and an operation to the left of the equal sign and the answer to the right of the equal sign. This may throw some students into disequilibrium. Asking students to choose whether each number sentence is true or false can encourage them to examine their assumptions about the equal sign.

Note: We are trying to help students understand that the equal sign signifies a relation between two numbers. It is sometimes useful to use words that express that relation more directly (e.g., "Nine is the same as 4 plus 5").

3. Including zero in a number sentence may encourage students to accept a number sentence in which more than one number appears after the equal sign. For example:

9 + 5 = 14	9 + 5 = 14 + 0	9 + 5 = 0 + 14	9 + 5 = 13 + 1

4. Open number sentences given after the corresponding true/false questions are a nice way to follow up on the ideas that came out of the discussion of the true/false number sentence. The question being asked is:

"What number can you put in the box to make this number sentence true?"

4 + 5 = ☐	9 = 4 + ☐	9 = ☐
4 + 5 = ☐ + 5	4 + 5 = ☐ + 4	4 + 5 = ☐ + 3
☐ = 4 + 5	4 + ☐ = 9	
2 x 4 = ☐	8 = 2 x ☐	8 = ☐
2 x 4 = ☐ x 4	2 x 4 = ☐ x 2	2 x 4 = ☐ x 8
☐ = 2 x 4	2 x ☐ = 8	
15 – 7 = ☐	8 = 15 – ☐	8 = ☐
15 – 7 = ☐ – 7	15 – 7 = ☐ – 8	☐ = 15 – 7
15 - ☐ = 8		
10 ÷ 2 = ☐	5 = 10 ÷ ☐	5 = ☐
10 ÷ 2 = ☐ ÷ 2	10 ÷ 2 = ☐ ÷ 3	☐ = 10 ÷ 2
10 ÷ ☐ = 5		

Scaffolding:

The following are benchmarks to work toward as children's conception of the equal sign evolves.

1. Getting children to be specific about what they think the equal sign means (even if their thinking is incorrect). To do this, the conversation must go beyond just comparing the different answers to the problem. For example, in the problem 8 + 4 = ☐ + 5, some children might say:

   The equal sign must be preceded by two numbers joined by a plus or a minus and followed by the answer (resulting in an answer of 12 to this problem).

   You have to use all the numbers (resulting in an answer of 17 to this problem).

   Though this understanding is not correct, the articulation of conceptions represents progress.

2. Children accept as true some of the number sentences that are not of the form a + b = c (e.g., 8 = 5 + 3; 8 = 8; 3 + 5 = 8 + 0; or 3 + 5 = 3 + 5).

3. Children recognize that the equal sign represents a relation between two equal numbers (rather than "the answer is"). Children might compare the two sides of the equal sign by carrying out the calculation on each side.

4. Children are able to compare the mathematical expression without actually carrying out the calculation. For example: 8 + 4 = □ + 5

A child might say, "*I saw that the 5 over here (pointing to the 5 in the number sentence) was one more than the 4 over here (pointing to the 4 in the number sentence), so the number in the box had to be one less than the 8. So it's 7."*

Guiding Questions:

- *Why do you think that?*
- *Why do you think that you cannot write number sentences that look like that?*
- *Do you agree or disagree with Student A? Why?*
- *Why do you believe this equation is true?*
- *Why do you believe this equation is false?*
- *What do you do when there is more than one number that follows the equal sign?*
- *How do you know that the number that you put in the box makes the number sentence true?*
- *How can you figure out the number that goes in the box without doing any calculation?*

Reference

Carpenter, Thomas P. Franke, Megan Loef, Levi, Linda, Thinking Mathematically: Integrating Arithmetic & Algebra in Elementary School, Portsmouth, N.H.: Heinemann, 2003.

Falkner, Karen P., Levi, Linda, & Carpenter, Thomas P. 1999. "Children's Understanding of Equality: A Foundation for Algebra." *Teaching Children Mathematics 6*, 232-236.