Grade 4

Number Strings

Purpose:

- To use number relationships to solve problems and to learn number facts
- To use known facts and relationships to determine unknown facts
- To develop and test conjectures
- To make generalizations about mathematical relationships, operations and properties

Description:

This routine focuses on developing a sense of pattern and relationships among related problems. The task is at a higher level than merely recalling basic facts. Students identify and describe number patterns and relationships within and among equations. Students make conjectures about the patterns and relationships they notice. During this process, students explain their reasoning. Over time, students develop generalizations about important number relationships, operations and properties. These generalizations assist in solving problems and learning number facts.

Materials:

- Prepared list of number strings
- Whiteboard, chart paper, or overhead transparencies
- Student journals, student whiteboards, or scratch paper

Time: 15 minutes maximum per session

Directions:

Example:
a. 2 x 5 =
b. 4 x 5 =
c. 8 x 5 =
d. 16 x 5 =
e. 32 x 5 =
f. 48 x 50 =

1. Write equation "a" and ask students to solve mentally (e.g., 2 x 5 =).
   Equation "a" should be easily accessible to all students.
2. Have students check their answer with a partner.
3. Ask one student to share his/her solution with the class. Write the answer on the board to complete the equation (2 x 5 = 10).
4. Students show thumbs up or thumbs down for agreement or disagreement.
   - If there is agreement, go to equation "b."
   - If there is disagreement, facilitate a class conversation around the strategies the

student(s) used to arrive at the answer. Allow students to revise answers.

5. Give the students problem "b" to solve mentally (e.g., 4 x 5). Repeat, #'s 2, 3, and 4 above.

6. Write problem "c" (e.g., 8 x 5). Ask students how they could use what they know about the first two equations to solve this equation. Partner talk.

7. A volunteer shares his/her mathematical reasoning that derived an answer to this equation (e.g., "I know that the factor '2' in the first equation was multiplied by 2 to get the new factor '4' in the second equation. The '5' stayed the same so the product was also multiplied by 2: 10 x 2 = 20. Since the '8' in the third equation is 2 four times and the '5' stayed the same, then the product should also be multiplied four times: 10 x 4 = 40").

   Note: If students are having difficulty sharing relationships, ask questions such as the following:
   - *How are equations "a" and "b" alike?*
   - *How are equations "a" and "b" different?*
   - *Describe the relationship between the factors?*
   - *Describe the relationship between the products?*
   - *How can we use these relationships to predict the product for equation "c?"*

8. Write problem "d" (e.g., 16 x 5). Ask students to predict their answer to this problem. Students share their predictions with their partner and explain their thinking. Teacher writes predictions on the board.

9. A volunteer shares his/her mathematical reasoning that derived the answer to this equation (e.g., "16 x 5 =   " could lead to a discussion about quadrupling the "4 x 5 =   " equation or doubling the "8 x 5 =   " equation.)

10. Repeat steps 8 and 9 for equations "e," "f," and "g."

   Note: When students get to an equation that does not necessarily follow the same pattern (e.g., doubling), the discussion should yield many different strategies. (e.g., "48 x 5 =   " could be solved by adding the products of 16 x 5 and 32 x 5, or by multiplying the product of 8 x 5 by 6, or by multiplying the product of 2 x 5 by 24, etc.)

11. When the string is completed, facilitate a conversation about how relating a known equation can help students solve unknown equations. Listen for what relationships students notice throughout the string and how students are able to extend patterns beyond the string you have written. Ask students to make statements about the patterns and/or relationships that helped them to complete the string.

12. Examine the "conjectures" that the students share. Ask questions such as:
   - *Will doubling one factor always result in a doubled product? How can you prove your conjecture?*
   - *Will this always work? How can you prove your conjecture?*

Scaffold:

Begin with strings that grow in a predictable way and are easily accessible to all students

Possible Number Strings:

2 x 5 =	1 x 10 =	1 x 12 =	8 x 1 =
4 x 5 =	2 x 10 =	2 x 12 =	8 x 2 =
8 x 5 =	3 x 10 =	3 x 12 =	8 x 3 =
16 x 5 =	4 x 10 =	4 x 12 =	8 x 4 =
32 x 5 =	5 x 10 =	6 x 12 =	8 x 8 =
48 x 5 =	6 x 10 =	8 x 12 =	8 x 10 =
48 x 50 =	6 x 20 =	8 x 120 =	4 x 10 =
	6 x 200 =	8 x 121 =	12 x 10 =

12 ÷ 12 =	36 ÷ 3 =	8 ÷ 2 =	14 ÷ 7 =
12 ÷ 6 =	36 ÷ 6 =	16 ÷ 2 =	140 ÷ 7 =
12 ÷ 4 =	18 ÷ 6 =	32 ÷ 2 =	280 ÷ 7 =
12 ÷ 3 =	180 ÷ 6 =	48 ÷ 2 =	287 ÷ 7 =
12 ÷ 2 =	180 ÷ 12 =	48 ÷ 4 =	280 ÷ 14 =
12 ÷ 1 =	1800 ÷ 12 =	480 ÷ 4 =	2800 ÷ 14 =
12 ÷ 1/2 =	3600 ÷ 12 =	484 ÷ 4 =	2814 ÷ 14
12 ÷ 1/4 =		480 ÷ 40 =	

Generalizations to Develop Through These Strings:

Note: ***Do not tell students these generalizations. Ask students to make conjectures first and then ask them to test their conjectures using three or more examples. If the conjectures always holds true, then the students can make "generalizations."***

In multiplication, many strings begin by doubling one factor while leaving the other factor the same (e.g., 2 x 5 becomes 4 x 5). This always doubles the product accordingly (e.g., 2 x 5 = 10 becomes 4 x 5 = 20). The Big Idea associated with this pattern is: **By whatever amount the factor is multiplied, the product will be multiplied by the same amount.**

In division, this relationship holds true with the dividend and the quotient as well. **As the dividend is doubled** (8 ÷ 2 becomes 16 ÷ 2), **the quotient is doubled accordingly** (8 ÷ 2 = 4 becomes 16 ÷ 2 = 8).

The divisor has an inverse (opposite) relationship with the quotient. **As the divisor is multiplied by an amount, the quotient is divided by that same amount** (e.g., 36 ÷ 3 = 12 becomes 36 ÷ 6 = 6).

Sometimes the pattern is predictable because a factor is being doubled over and over, so the product doubles over and over, as well. But then, the pattern may change (e.g., 8 x 5 = 40, 16 x 5 = 80, 32 x 5 = 160, then 48 x 5 = ?).

In order to make sense of this situation a student must understand the associated Big Idea: **Numbers are the sum of more than one quantity** (e.g., 48 = 16 + 32). **The Distributive Property states that when a number is being multiplied by a particular factor, it is equivalent to multiplying the number by the parts that make up that factor** [e.g., 48 x 5 = (16 x 5) + (32 x 5)].

This Big Idea can help students develop an understanding of the relationships among numbers that will aid them in finding unknown products by relying on known facts (see *Using Strings to Learn Multiplication Facts* below).

For example:
Because 48 = 16 + 32, and students already know what 16 x 5 and 32 x 5 are, they can derive 48 x 5 as follows:

48 x 5 = (16 x 5) + (32 x 5)
240 80 + 160

The Distributive Property also states that when a dividend is being divided by a particular divisor (e.g., 2814 ÷ 14)**, it is equivalent to dividing the parts that make up that dividend by the same divisor and then adding the quotients**
[e.g., 2814 ÷ 14 = (2800 ÷ 14) + (14 ÷ 14)].

For Example:
Because 2814 = 2800 + 14, and students already know that 2800 ÷ 14 = 200 and 14 ÷ 14 = 1, they can derive 2814 ÷ 14 as follows:

2814 ÷ 14 = (2800 ÷ 14) + (14 ÷ 14)
201 200 + 1

Using Strings to Learn Multiplication Facts

Strings can be helpful to assist students to learn their multiplication facts as they learn to see the relationships among the facts.

Example I: If a student cannot remember 8 x 6, but knows 4 x 6, all the student has to do is double the product of 4 x 6 because 8 = 2 x 4.

4 x 6 = 24
8 x 6 = 48

Example II: If a student cannot remember 8 x 6, but knows 2 x 6 and 6 x 6, all the student has to do is find the product of these two equations and then find the sum of the products because 8 = 2 + 6.

2 x 6 = 12 6 x 6 = 36 8 x 6 = 48

Guiding Questions:

- What pattern(s) do you see?
- What stayed the same?
- What changed?
- How did it change?
- How did knowing the answers to the first equation help you figure out the answer to the next equation?
- Does this always work? How do you know?
- How are equations “a” and “b” alike?
- How are equations “a” and “b” different?
- What is the relationship between the factors?
- What is the relationship between the products?
- How can we use these relationships to predict the product for equation “c?”
- What is the relationship between the dividends?
- What is the relationship between the divisors?
- How can we use these relationships to predict the quotient for equation “c?”
- What is the relationship between the quotients?