Grade 4

Thinking Relationally

Purpose:

- To make the learning of arithmetic richer
- To think flexibly about mathematical operations
- To compare mathematical expressions without actually carrying out the calculation
- To help students recognize without having to calculate that the expressions on each side of the equal sign represent the same number
- To provide a foundation for smoothing the transition to algebra

  Note: In algebra, students must deal with expressions that involve adding, subtracting, multiplying, and dividing but that are not amenable to calculation (e.g., $3x + 7y – 4 z$). They have to think about relations between expressions ($5x + 34 = 79 – 2x$) as they attempt to figure out how to transform equations in order to solve them.

Description:

Students are engaged in conversations about the relationships between numbers and how these relationships can be useful in finding solutions to problems. Students analyze expressions through the context of true/false and open number sentences. Students find ways to solve the problems by using number relations before calculating the answers.

Materials:

Purposely planned equations.

Note: Select equations that cannot be easily calculated. We want students to be motivated to look for relations. If equations can be easily calculated, the need does not exist to look for number relations.

Time: 15 minutes maximum

Directions:

1. To successfully implement relational thinking routines, the following classroom norms must be established:

- Students explain their thinking
- Students listen to one another
- Alternative strategies for solving a given problem are valued and discussed
- Solutions that involve more than simply calculating answers are not only accepted, but valued

2. You, the teacher, will need to make decisions based on the needs of your class. As you select problems:

   - Start with relatively easy problems and selected problems that provide an appropriate level of challenge based on what you have observed students doing on previous problems.
   - Select problems that will challenge students but not be too difficult for them.
   - Make decisions about what problems to use next based on students' responses to problems that they had already solved.

3. **Goal 1: For students to recognize that they do not always need to carry out calculations; they can compare expressions before they calculate.**

   Engage students in a general discussion about what it means for a number sentence to be true or false. Pose the following true/false problems (one at a time):

   12 – 9 = 3
   34 – 19 = 15
   5 + 7 = 11
   58 + 76 = 354

   Students explain how they know whether the number sentence is true or false. Students justify their solutions with their partner.
   Notice which students are calculating and which students are using relationships to determine whether the problems are true or false.

4. Pose the following true/false problem:

   27 + 48 - 48 = 27

   Students justify their answers.
   This problem establishes that students do not necessarily have to calculate to decide if a number sentence is true or false.

5. Ask students to see if they can figure out in their heads whether the following problem is true or false (without adding or subtracting):

   $$48 + 63 – 62 = 49$$

   Students justify their answer.
   This problem extends the idea that was used in the previous problem.

6. Pose the following true/false problem:

   $$674 + 56 – 59 = 671$$

   Students justify their solutions.

   This problem is slightly more complicated than the preceding problem because students have to recognize that 59 breaks apart to 56 + 3 and that they can subtract 56 from the 56 given in the problem, and then they have to subtract 3 more from 674.

7. **Goal 2: To use properties of numbers and operations to think about relations between numerical expressions.**

   Review open number sentences. Pose the following problem:

   What number would you put in the box to make this a true number sentence?

   $$7 + 6 = \square + 5$$

   Students justify their solutions.

8. Pose the following problems (one at a time):

   $$43 + 28 = \square + 42$$
   $$28 + 32 = 27 + \square$$
   $$67 + 83 = \square + 82$$

   Students justify their solutions.
   Children must recognize that they can use relational thinking to solve these problems without carrying out all the calculations.

9. Up until this point, boxes have been used to represent an unknown in an open number sentence. Students readily adapt to using letters to represent variables and unknowns. Pose the following problem:

$$12 + 9 = 10 + 8 + c$$

What is the value of c?

Students justify their solutions.
If students justify their answers with an explanation focusing on computation, ask how this problem could be solved without adding 12 + 9 or 10 + 8 (e.g., 10 is two less than 12 and eight is one less than nine, so c must be 3).

10. Pose a problem with larger numbers but the same general structure, as follows:

$$345 + 576 = 342 + 574 + d$$

What is the value of d?

Students justify their solutions.

11. Pose the following problem:

$$46 + 28 = 27 + 50 – p$$

What is the value of p?

Students justify their solutions.

12. When students have figured out how to deal with addition problems, move to subtraction problems. Pose the following problem:

$$86 – 28 = 86 – 29 – g$$

What is the value of g?

13. **Goal 3: Using relational thinking to learn multiplication facts**

The following problems can be used to draw children's attention to relations among numbers that can make learning number facts easier.

- Knowing that addition and multiplication are commutative reduces the quantity of number facts that children have to learn by almost half.

True/False

3 x 4 = 4 x 3

What number would you put in the box to make this a true number sentence?

4 x 8 = 8 x ☐

- Understanding the relation between addition and multiplication makes it possible for students to relate the learning of multiplication facts to their knowledge of addition.

True/False

3 x 7 = 7 + 7 + 7
3 x 7 = 14 + 7
4 x 6 = 12 + 12
6 x 4 = 4 + 4 + 4 + 4 (false)

- Focusing on specific relationships among multiplication facts can make it possible for students to build on the facts they have learned.

True/False

3 x 8 = 2 x 8 + 8
6 x 7 = 5 x 7 + 7
8 x 6 = 8 x 5 + 6 (false)
7 x 6 = 7 x 5 + 7
9 x 7 = 10 x 7 - 7

Sample problems to assist in developing relational thinking

True/False (not all are true)

37 + 56 = 39 + 54	37 x 54 = 38 x 53
33 – 27 = 34 – 26	60 x 48 = 6 x 480
471 – 382 = 474 – 385	5 x 84 = 10 x 42

674 – 389 = 664 – 379

583 – 529 = 83 – 29

64 ÷ 14 = 32 ÷ 28

42 ÷ 16 = 84 ÷ 32

Sample problems for developing understanding of the properties of numbers and operations within numerical expressions

73 + 56 = 71 + d

68 + b = 57 + 69

96 + 67 = 67 + p

87 + 45 = y + 46

92 – 57 = g – 56

56 – 23 = f – 25

74 – 37 = 75 – q

73 + 56 = 71 + 59 – d

68 + 58 = 57 + 69 – b

96 + 67 = 67 + 93 + p

87 + 45 = 86 + 46 + t

92 – 57 = 94 – 56 + g

56 – 23 = 59 – 25 – s

74 – 37 = 71 – 39 + q

Sample problems for developing base ten concepts

True/False (not all are true)

56 = 50 + 6

87 = 7 + 80

93 = 9 + 30

94 = 80 + 14

94 = 70 + 24

246 = 24 x 10 + 6

47 + 38 = 40 + 7 + 30 + 8

24 + 78 = 78 + 20 + 2 + 2

63 – 28 = 60 – 20 – 3 – 8

63 – 28 = 60 – 20 + 3 – 8

.78 = 7.8

1.95 = 1.9500

Reference

Carpenter, Thomas P. Franke, Megan Loef, Levi, Linda, Thinking Mathematically: Integrating Arithmetic & Algebra in Elementary School, Portsmouth, N.H.: Heinemann, 2003.